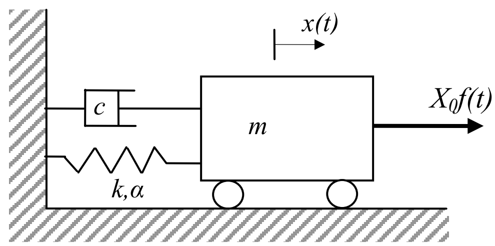
NEEP 602 -- Engineering Problem Solving II
Homework 3
Due Thursday 3/3/05

6. Nonlinear Spring-Mass-Damper Model – Using Excel (20 points)

The system shown in the figure is a typical model used to study the response of a variety of mechanical systems subjected to an excitation $X_0 f(t)$. The spring in this system happens to be a nonlinear cubic spring.



The solution to the second-order differential equation of motion given below describes the displacement x(t), where $\tau = \omega_n t$ is the nondimensional time , $\zeta = \sqrt{k/m}$ is the damping coefficient, and α is the coefficient of nonlinearity.

$$\frac{d^2x}{d\tau^2} + 2\zeta\frac{dx}{d\tau} + x + \alpha x^3 = X_0 f(\tau)$$

Solve the equation using a 4^{th}-order Runge Kutta algorithm in Excel for $\tau = 0$ to 40, for the three cases given in the table. The first case is a linear system, to permit comparison with the nonlinear cases.

Case	type	α	ζ	x(0)	$\frac{dx}{d\tau}(0)$
1	linear	0.00	0.20	-2.00	2.00
2	nonlinear	-0.25	0.20	-2.00	2.00
3	nonlinear	-0.25	0.20	-2.00	2.31

Plot the displacement x of the mass m vs. time (τ) for the three cases, and also the phase plots. To facilitate comparison of the different cases, you should produce only two charts: one with the x vs. τ curves and the second with the phase plots ($dx/d\tau$ vs. x). Comment briefly on how the motion for each system varies with time, and whether or not the motions of the linear and nonlinear systems converge with or diverge from one another as time goes on.

7. Crane with Swinging Load – Using Excel (15 points)

The overhead crane trolley is transporting a load *m*, supported by a cable of length *L* as shown in the figure. When the trolley is subjected to an acceleration *a(t)*, the equation of motion for the load is

$$L\frac{d^2\theta}{dt^2} + g\sin\theta = -a(t)\cos\theta - c\dot{\theta}$$

where g =9.81 m/s^2 and c is the damping coefficient.

[Figure cut and pasted in the old-fashioned way]

For a cable length of L = 2 m and damping coefficient c = 0.75, produce a graph that shows the motion θ and $d\theta/dt$ for the load if $\theta(0)$ = 0.2 rad, $d\theta(0)/dt$ = 0 rad/s and $a(t) = 5u(t)$m/s^2 for $0 \le t \le 5$s (here $u(t)$ is the unit step function) and $a(t) = 0$ for $5 < t \le 20$s.