NEEP 602 -- Engineering Problem Solving II
Homework 1
Due Thursday 2/3/05

1. Error Function (10 points)

The error function *erf(x)* is usually defined in terms of the integral

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$$

but it is also the solution to the differential equation

$$\frac{dy}{dx} = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

$$y(0) = 0$$

Use Matlab solvers `ode45` to solve this initial value problem on the interval $0 \le x \le 2$ and compare your results with the Matlab function `erf(x)`.

2. Body Falling Through Oil (15 points)

An object falling through a dense, viscous fluid like oil is acted on by three forces: a resistive, or drag, force, a buoyant force, and a weight force due to gravity. For a spherical body of radius *a*, Stokes' Law gives the drag force (if the velocity is not too large) as

$$R = 6\pi\mu a v$$

where μ is the dynamic viscosity and v is the velocity. The buoyant force is of course equal to the weight of the fluid displaced by the object.

Write a differential equation that can be solved to show how the velocity of a steel sphere falling through an oil bath will vary with time *t*. Use Matlab to solve the differential equation and plot *v* vs. *t*. Determine the terminal velocity of the sphere and have your script display that result. Steel has a specific weight of 0.284 lb/in^3, and the oil has a dynamic viscosity of 400 centipoise and a density of 890 kg/m^3. The sphere is 14mm in diameter, and it starts from rest.

3. Satellite Orbits (15 points)

The equations of motion for a satellite or moon in orbit around a planet are as follows:

$$\frac{d^2 r}{d\tau^2} - r\left(\frac{d\theta}{d\tau}\right)^2 = -\frac{4\pi^2}{r^2}$$

$$r\frac{d^2\theta}{d\tau^2} + 2\frac{dr}{d\tau}\frac{d\theta}{d\tau} = 0$$

Here $\tau = t/P_c$, t is time, and P_c is the period of a circular orbit at the planet's surface; the equations as given above can then be solved to show the shape and relative size of various orbits, depending on the initial conditions. Convert the two second order differential equations into a set of four coupled first order equations, and use `ode45` to generate a polar plot of the orbits resulting from each of the three sets of initial conditions given in the table.

Set	***r(0)***	***dr/dτ(0)***	***θ(0)***	***dθ/dτ(0)***
1	2	0	0	1.5
2	1	0	0	2π
3	2	0	0	4

Show all three orbits on one plot, and label them according to the type of orbit (circular, elliptical, or hyperbolic).