NEEP 602 -- Engineering Problem Solving II
Spring 2005
Exercise 2

More with Lorenz equations

Lorenz simplified a complicated model of the weather down to a system of three equations:

$$\frac{dx}{dt} = -3(x - y)$$
$$\frac{dy}{dt} = -xz + rx - y$$
$$\frac{dz}{dt} = xy - z$$

For r = 8 this system approaches a stable equilibrium, while for r = 26 it exhibites chaotic behavior.

One way to explore this behavior is to plot phase plots for x, y, and z. you can do this in both 2-D and 3-D using the following commands:

```
time=[0 40];
init=[1 1 1];
[t w]=ode45('rkfunc', time, init);
plot(t,w)
pause
x=w(:,1);
y=w(:,2);
z=w(:,3);
plot(x,y)
pause
plot(x,z)
pause
plot(z,y)
pause
plot3(x,y,z)
```

There are 3 two-dimensional phase plots and the last plot is a phase plot in 3-D. Notice how for r=8, the system approaches a stable equilibrium, while for r=26, the system is chaotic and the solution revolves bounces between the two possible stable points.

This system also exhibits a sensitivity to initial conditions. That is, a slight change in initial conditions causes a significant change in the solution. Try solving the system for r=26 on a time interval $0<t<30$ using the following commands:

```
[t w]=ode45('rkfunc', [0 30], [1 1 1.001]);
hold on
plot(t,w)
[t w]=ode45('rkfunc', [0 30], [1 1 1]);
plot(t,w,'c')
```

The last command plots in a light blue (cyan). Notice how the two plots diverge after about 15 seconds.

Second-order equations

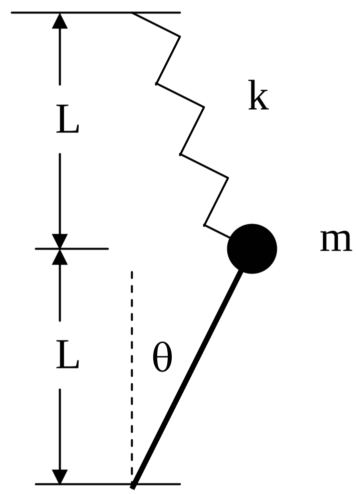
To solve second order equations, we convert them to a system of two first order equations, as described in exercise 1, and then solve the system. Suppose we want to model a damped pendulum. The differential equation and initial conditions are:

$$\frac{d^2\theta}{dt^2} + \alpha\dot{\theta} + \sin(\theta) = 0$$
$$\theta(0) = A$$
$$\frac{d\theta}{dt}(0) = 0$$

- Solve this system for $A=\pi/4$ and various values of α and plot the phase portraits for the solutions. Choose $0<\alpha<0.25$.
- What happens when $\alpha=0$ and the initial angular velocity is 2 radians per second?

An Example – The Inverted Pendulum

Consider an inverted pendulum connected to a fixed point by a spring, as shown in the figure below.



The governing equation for this system is:

$$\frac{d^2\theta}{dt^2} = \sin(\theta) - \frac{2kL}{mg}\sin(\theta)\left(1 - \frac{1}{\sqrt{5-4\cos(\theta)}}\right)$$

Assuming an intial angle of $\theta=\pi/4$, find combinations of the parameters that 1) cause an initial increase in the angle of the pendulum, 2) cause an initial decrease in the angle of the pendulum, and 3) lead to no motion of the pendulum.