Physics 525 Take Home Final Examination

Due 5/12/2004 in 1422 Engineering Hall by 9:45 AM (3 pages, 6 problems)
You may use any resources you may find helpful except the assistance of your classmates or other persons. Clearly state any assumptions you make in solution of the problems.
Please write clearly!

1) (10 pts) Discuss the Boltzmann distribution of electron density with respect to potential. Give a physical interpretation. When can we use it? When can we not use it? Where do we need to be careful? Give specific examples as appropriate.

2) (15 pts) The solar wind is an interplanetary plasma that streams from the sun's atmosphere, and "carries" with it an interplanetary magnetic field (also from the sun). At the location of the earth's orbit, the proton density is approximately $4 \times 10^6\ m^{-3}$, the typical wind speed of the protons is 400 km/sec, and the interplanetary magnetic field is 5×10^{-9} T. These values are characteristic of a "quiet" sun. This wind strikes the magnetic field of the earth and distorts it.

(a). What is the kinetic particle pressure of the solar wind (its so-called "ram" pressure)?

(b). What is the distance (measured in earth radii) from the center of the earth in the equatorial plane at which the magnetic pressure of the earth's magnetic field is equal to the ram pressure of the solar wind? This boundary is known as the magnetopause. This distance marks one boundary of the magnetosphere inside which the nearly static plasma of the earth's upper atmosphere is contained. Outside this envelope, the solar wind is deflected by the earth's magnetic field, and flows around the magnetosphere. Take the equatorial magnetic field to be $B = 3.1 \times 10^{-5}$ T at the **surface** of the earth (on the equator).

(c). Is the solar wind's magnetic field important in this pressure balance? Why or why not?

(d) Occasionally the sun belches out massive quantities of plasma in what are called mass coronal ejections. In this case the solar wind can increase dramatically. Communication satellites are located in 'geo-synchronous' orbits at a distance of ~25,000 miles altitude. How much stronger would the solar wind (ram pressure) need to get to move the magnetopause inside of the geo-synchronous orbit and severely disrupt satellite communications? (Use 4000 mi for radius of Earth)

3) (15 pts) Alfven waves from MHD. We derived Alfven waves using a 2-fluid model in class. This can be done very simply using the ideal MHD equations. Consider a uniform plasma with a uniform magnetic field. Using

$$r \frac{dv}{dt} = \underline{J} \times \underline{B} \quad \text{and} \quad \underline{E} + \underline{v} \times \underline{B} = 0$$

together with Amperes Law and Faradays Law derive the usual dispersion relation for Alfven waves

$$\boxed{\frac{w}{k} = V_A}$$

What assumptions are in this derivation?

4) (15 pts) One scheme for generating electric power is to collect sunlight using a satellite with solar cells, convert the energy to microwaves, and transmit the energy by electromagnetic waves to a receiving station on the earth. To get to the earth, the microwaves must traverse the ionosphere, where the plasma there may absorb some energy due to collisions between the plasma electrons and neutral gas. Take the average plasma density of the ionosphere to be $n_e = 10^{11}$ m^{-3}, and the electron neutral collision frequency to be $\nu = 1000$ s^{-1}.

(a). Recall that for an electromagnetic wave propagating through a plasma (assuming no magnetic field)

$$w^2 = w_{pe}^2 + c^2k^2$$

For a microwave frequency f = 100 MHz = 10^8 Hz, what is the wavelength λ of the wave in the ionospheric plasma?

(b). Fortunately, you have already derived (HW) the dispersion relation taking into account electron-neutral collisions, and know that for $k_I << k_R$:

$$k_I = \frac{n}{2c}\frac{w_{pe}^2}{w^2}\left(1 - \frac{w_{pe}^2}{w^2}\right)^{-1/2}$$

After traversing a 200 km thickness of the ionospheric plasma, by what fraction has the wave amplitude decreased? Since wave power is proportional to the square of the amplitude of the wave, what fraction of the microwave power is lost as it crosses the ionosphere?

(c). Is this a good idea? Discuss at least three possible problems/difficulties with this approach.

5) (15 pts) The Department of Homeland Security decided to implement a space-spaced laser to shoot down incoming missiles that might be launched by terrorist (pick your 'favorite') groups. They decided to use a Nd:YAG based system (because very high pulsed powers from solid-state systems could be achieved) operating at 1064 nm (free-space wavelength). The terrorists, having taken Physics 525, realize that there may be an easy defense to this weapon. They coat the warheads with a carbon layer, knowing that when the laser first hits it will eject carbon atoms and form a plasma. Assume that this plasma that is formed is made up of triply ionized carbon, C^{+++} and a sufficient number of electrons to maintain quasi-neutrality:

a) What is the ratio of the electron density to the carbon density?
b) What must the carbon density be if the rest of the laser pulse is to be reflected off the initially formed plasma layer? Assume normal incidence.
c) The warhead could produce its own shield through friction with the air. How hot would the carbon gas ablating from around the warhead need to be to form an initial 80% singly-ionized carbon plasma with the carbon neutral density calculated in part b? (C^+ ionization energy 11.26 eV)?

I could have added to this to figure out how thick the carbon layer needed to be for a given ablation/depletion rate and flight time and thermal transfer rates to the delicate electronics inside the warhead, but…. You people need a break!

6) (30 pts) Consider a cylindrically symmetric plasma contained by an axial magnetic field. The column has a length L and radius a.

(a). For $kT_e = kT_i = 1$ keV (uniform), $n_e(r) = n_{eo}(1-r^2/a^2)$; $n_{eo} = 2 \times 10^{19}\ m^{-3}$, $B_0 = 1.0$ T, L = 2.0 m, a = 0.2 m, find the value of β at r = 0, and the total stored energy in the plasma.

Now, we introduce a current I in the z-direction, with uniform current density $\vec{J} = J_0\hat{z}$. This is a hydrogen plasma. This current maintains the parameters and profiles in part (a).

(b) What is the magnitude of this current for the plasma to be in MHD equilibrium?

(c). What is the magnetic field $\vec{B}(r)$ produced by this plasma **current**? (Be sure to give the direction.)

(d). Derive an expression for the $\nabla|B|$ drift of a charged particle with $v_\parallel = 0$ in terms of B_o, J_0, r, $KT_\perp$, e, m, etc. Assume that the field you found in part (c) is small compared to B_0, but <u>not</u> zero. Recall that the $\nabla|B|$ depends on the gradient of the magnitude of the <u>total</u> field.

(e). Because the electron temperature is not infinite, the plasma has a non-zero resistivity. What is the electric field $\vec{E} = E_0\hat{z}$ that is required to produce the steady current I?

(f). From the global power balance (stored energy and input power), deduce the energy confinement time for this system. Comment on it and what we have left out in this calculation.