$$0 = -\frac{dp}{dz} + \mu \frac{d^2 u_z}{dy^2}$$

The pressure difference can be written as

$$\frac{dp}{dz} = -\frac{\Delta p}{L}$$ (This is because pressure is decreasing)

The momentum eqn. now becomes:

$$0 = -\left(-\frac{\Delta p}{L}\right) + \mu \frac{d^2 u_z}{dy^2} = \frac{\Delta p}{L} + \mu \frac{d^2 u_z}{dy^2}$$

Rearranging

$$-\frac{\Delta p}{\mu L} = \frac{d^2 u_z}{dy^2}$$

Integrate once

$$-\frac{\Delta p}{\mu L} y + C_1 = \frac{du_z}{dy}$$

Replace B.C (b) $\frac{du_z(y=0)}{dy} = 0$

$$C_1 = 0$$

$$\frac{du_z}{dy} = -\frac{\Delta p}{\mu L} y$$

Integrate

$$u_z = -\frac{\Delta p}{2\mu L} y^2 + C_2$$

Replace B.C (a) $u_z(y = h/2) = 0$

$$C_2 = \frac{\Delta p}{8\mu L} h^2$$

And the velocity is

$$u_z = \frac{\Delta p h^2}{8\mu L}\left(1 - \left(\frac{2y}{h}\right)^2\right)$$