ME 240 EQUATIONS

PARTICLE DYNAMICS

General Expressions

$$v = \frac{ds}{dt}$$
$$a = \frac{dv}{dt}$$
$$a = v\frac{dv}{ds}$$

Constant Acceleration

$$v = v_o + a_c t$$
$$s = s_o + v_o t + \frac{1}{2}a_c t^2$$
$$v^2 = v_o^2 + 2a_c(s - s_o)$$

Relative Motion

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$
$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

Normal & Tangential Components

$$\vec{v} = v\vec{e}_t$$
$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n$$

Radial & Transverse Components

$$\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

Newton's Law

$$\sum \vec{F} = m\vec{a}$$

Newton's Gravitational Law

$$\sum \vec{F} = \frac{GMm}{r^2}$$

Work and Energy Relationships

$$T = \frac{1}{2}mv^2$$
$$U_{1\to 2} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$
$$U_{1\to 2} = \int_{s_1}^{s_2} -ksds \quad (spring)$$
$$U_{1\to 2} = -W(y_2 - y_1) \quad (gravity)$$
$$T_2 = T_1 + \sum U_{1\to 2}$$
$$T_1 + V_1 = T_2 + V_2$$
$$Power = \frac{dU}{dt}$$
$$V_e = \frac{1}{2}k(s - s_o)^2 \quad (spring)$$
$$V_g = Wy \quad (gravity)$$

Impulse and Momentum Relationships

$$\vec{L} = m\vec{v}$$
$$\vec{H}_o = \vec{r} \times m\vec{v}$$
$$\vec{M}_o = \vec{r} \times \vec{F}$$
$$\sum (m\vec{v})_2 = \sum (m\vec{v})_1 + \sum \int \vec{F}dt$$
$$e = \frac{v'_B - v'_A}{v_A - v_B} = \frac{-(v'_{A/B})}{(v_{A/B})}$$

RIGID BODY DYNAMICS

General Expressions

$$\omega = \frac{d\theta}{dt}$$
$$\alpha = \frac{d\omega}{dt}$$
$$\alpha = \omega\frac{d\omega}{d\theta}$$

Constant Angular Acceleration

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$
$$\omega = \omega_o + \alpha t$$
$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

Relative Motion

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$
$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

General Plane Motion

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

Relative Motion with Rotating Axes

$$\vec{v}_B = \vec{v}_A + \vec{\Omega} \times \vec{r}_{B/A} + \vec{v}_{Brel}$$
$$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}_{B/A}\right) + 2\vec{\Omega} \times \vec{v}_{Brel} + \vec{a}_{Brel}$$

Mass Moments of Inertia

$$\bar{I} = I_G = \frac{1}{2}mr^2 \quad (disk)$$
$$\bar{I} = I_G = \frac{1}{12}ml^2 \quad (rod)$$
$$\bar{I} = I_G = \frac{2}{5}ma^2 \quad (sphere)$$

$$I_o = I_G + md^2$$
$$\bar{I} = m\bar{k}^2$$

Newton's Law

$$\sum \vec{F} = m\vec{a}_G$$
$$\sum \vec{M}_G = \bar{I}_G\vec{\alpha}$$

Work and Energy

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$
$$U_{1\to2} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$
$$U_{1\to2} = \int_{\theta_1}^{\theta_2} \vec{M} \cdot d\vec{\theta}$$
$$T_2 = T_1 + \sum U_{1\to2}$$
$$T_1 + V_1 = T_2 + V_2$$
$$Power = \frac{dU}{dt}$$

Impulse and Momentum Relationships

$$\vec{L} = m\vec{v}$$
$$\vec{H} = \vec{r} \times m\vec{v}$$
$$\vec{M} = \vec{r} \times \vec{F}$$
$$\vec{H}_A = \bar{r} \times m\bar{v} + \bar{I}\omega$$
$$\text{system momenta}_2 = \text{system momenta}_1 + \text{system ext. impulse}_{1\to2}$$
$$e = \frac{v'_B - v'_A}{v_A - v_B} = \frac{-(v'_{A/B})}{(v_{A/B})}$$