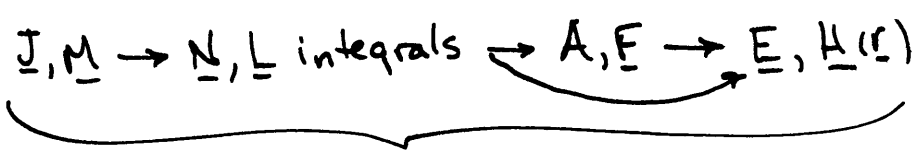


● The subject: radiated  $\underline{E}, \underline{H}$  (far field) from sources.

- Sources may include:
- currents on "antennas"
  - apertures, e.g., waveguide
  - scattering surfaces

Analysis "tool":



(formulation restricted to far field, where  $r \geq 2D^2/\lambda_0$ ,  $D > \lambda_0$ .)

Next: methods, theorems, even "tricks" to help implement the tool to solve as large a variety of problems as possible. This brings us to ...

Electromagnetic Theorems & Principles (Ch. 7)

- e.g., Duality, Uniqueness, Image Methods, Reciprocity, Reaction Equivalence, Induction Theorem, Physical Optics Theorem.

● One objective: substitute "actual" problem with an "equivalent" problem that is easier to solve, better suited to our "tool", or has already (previously) been solved.

(b)

## SCALING THEOREM\*

$$\nabla \times \underline{E} = -j\omega \underline{B} \quad \nabla \times \underline{H} = \underline{J} + j\omega \underline{D} \quad \nabla \cdot \underline{D} = \rho_e \quad \nabla \cdot \underline{B} = 0$$

$$\underline{D} = \epsilon \underline{E} \quad \underline{B} = \mu \underline{H} \quad \underline{J} = \sigma \underline{E}$$

Maxwell's Equations are invariant to the following size/wavelength transformation ( $K_i =$  scaling constant):

$$\boxed{l = K_1 l' \quad \omega = \frac{1}{K_2} \omega' \quad \underline{E} = K_3 \underline{E}' \quad \underline{H} = K_4 \underline{H}'}$$

unprimed = actual  
primed = transformed  
for expt. or  
modeling  
purposes

$$\underline{D} = \frac{K_4 K_2}{K_1} \underline{D}' \quad \underline{B} = \frac{K_3 K_2}{K_1} \underline{B}' \quad \underline{J} = \frac{K_4}{K_1} \underline{J}'$$

$$\rho_e = \frac{K_4 K_2}{(K_1)^2} \rho_e' \quad \epsilon = \frac{K_4 K_2}{K_3 K_1} \epsilon' \quad \mu = \frac{K_3 K_2}{K_4 K_1} \mu' \quad \sigma = \frac{K_4}{K_3 K_1} \sigma'$$

Thus, it is possible to make measurements in one system (the model,  $l', \omega'$ ) and to interpret them in terms of another system (the actual problem,  $l, \omega$ ) with different size & EM parameters & excited at a different frequency!

Examples:  $\boxed{l > l' \quad (K_1 > 1)$   
 $\omega < \omega' \quad (K_2 > 1)$

Scale down in size & up  
in frequency  
(lab models of RF in ionosphere)

$\boxed{l < l' \quad (K_1 < 1)$   
 $\omega > \omega' \quad (K_2 < 1)$

Scale up in size & down in freq.  
(microwave models of optical devices)

\* IEEE Trans. Ant & Prop. vol. 48, 1367-1375 (2000), by E. Behar.

# DUALITY PRINCIPLE

Idea: we have 2 sets of equations that are mathematically identical. Then, if you already have the solution to one set, you can easily get a solution to the 2nd (other) set by swapping "dual" quantities.

## Dual Equations

$$\boxed{\underline{J} \neq 0}$$

$$\nabla^2 \underline{A} + \beta^2 \underline{A} = -\mu \underline{J}$$
$$\underline{A}(\underline{r}) = \frac{\mu}{4\pi} \iiint_V \frac{\underline{J}(\underline{r}') e^{-j\beta R}}{R} d^3r'$$

$$(\underline{F} = 0)$$

$$\nabla \times \underline{E}_A = -j\omega \mu \underline{H}_A$$

$$\nabla \times \underline{H}_A = \underline{J} + j\omega \epsilon \underline{E}_A$$

$$(\underline{E}_F = \underline{H}_F = 0)$$

$$\underline{H}_A = \frac{1}{\mu} \nabla \times \underline{A}$$

$$\underline{E}_A = -j\omega \underline{A} - \frac{j}{\omega \mu \epsilon} \nabla (\nabla \cdot \underline{A})$$

$$\underline{E} = \underline{E}_A$$

$$\underline{H} = \underline{H}_A$$

$$\boxed{\underline{M} \neq 0}$$

$$\nabla^2 \underline{F} + \beta^2 \underline{F} = -\epsilon \underline{M}$$

$$\underline{F}(\underline{r}) = \frac{\epsilon}{4\pi} \iiint_V \frac{\underline{M}(\underline{r}') e^{-j\beta R}}{R} d^3r'$$

$$(\underline{A} = 0)$$

$$\nabla \times \underline{H}_F = +j\omega \epsilon \underline{E}_F$$

$$\nabla \times \underline{E}_F = -\underline{M} - j\omega \mu \underline{H}_F$$

$$(\underline{E}_A =$$

$$\underline{E}_F = -\frac{1}{\epsilon} \nabla \times \underline{F}$$

$$\underline{H}_F = -j\omega \underline{F} - \frac{j}{\omega \mu \epsilon} \nabla (\nabla \cdot \underline{F})$$

$$\underline{E} = \underline{E}_F$$

$$\underline{H} = \underline{H}_F$$