- The subject: radiated $\underline{E}, \underline{H}$ (far field) from sources.

Sources may include:
- currents on "antennas"
- apertures, e.g., waveguide
- scattering surfaces

Analysis "tool":

$$\underbrace{\underline{J}, \underline{M} \rightarrow \underline{N}, \underline{L} \text{ integrals} \rightarrow \underline{A}, \underline{F} \rightarrow \underline{E}, \underline{H}(\underline{r})}_{\text{(formulation restricted to far field, where } r \geq 2D^2/\lambda_0 \text{, } D > \lambda_0\text{)}}$$

Next: methods, theorems, even "tricks" to help implement the tool to solve as large a variety of problems as possible. This brings us to ...

Electromagnetic Theorems & Principles (Ch. 7)

e.g., Duality, Uniqueness, Image Methods, Reciprocity, Reaction Equivalence, Induction Theorem, Physical optics Theorem.

- One objective: substitute "actual" problem with an "equivalent" problem that is easier to solve, better suited to our "tool", or has already (previously) <u>been</u> solved.

SCALING THEOREM*

$$\nabla \times \underline{E} = -j\omega \underline{B} \qquad \nabla \times \underline{H} = \underline{J} + j\omega \underline{D} \qquad \nabla \cdot \underline{D} = \rho_e \qquad \nabla \cdot \underline{B} = 0$$

$$\underline{D} = \epsilon \underline{E} \qquad \underline{B} = \mu \underline{H} \qquad \underline{J} = \sigma \underline{E}$$

Maxwell's Equations are invariant to the following size/wavelength transformation (K_i = scaling constant):

$$\boxed{\ell = K_1 \ell' \qquad \omega = \frac{1}{K_2}\omega' \qquad \underline{E} = K_3 \underline{E}' \qquad \underline{H} = K_4 \underline{H}'}$$

unprimed = actual

primed = transformed for expt. or modeling purposes

$$\underline{D} = \frac{K_4 K_2}{K_1}\underline{D}' \qquad \underline{B} = \frac{K_3 K_2}{K_1}\underline{B}' \qquad \underline{J} = \frac{K_4}{K_1}\underline{J}'$$

$$\rho_e = \frac{K_4 K_2}{(K_1)^2}\rho_e' \qquad \epsilon = \frac{K_4 K_2}{K_3 K_1}\epsilon' \qquad \mu = \frac{K_3 K_2}{K_4 K_1}\mu' \qquad \sigma = \frac{K_4}{K_3 K_1}\sigma'$$

Thus, it is possible to make measurements in one system (the model, ℓ', ω') and to interpret them in terms of another system (the actual problem, ℓ, ω) with different size & EM parameters & excited at a different frequency!

Examples:

$\boxed{\begin{array}{l} \ell > \ell' \ (K_1 > 1) \\ \omega < \omega' \ (K_2 > 1) \end{array}}$

Scale down in size & up in frequency (lab models of RF in ionosphere)

$\boxed{\begin{array}{l} \ell < \ell' \ (K_1 < 1) \\ \omega > \omega' \ (K_2 < 1) \end{array}}$

Scale up in size & down in freq. (microwave models of optical devices)

* IEEE Trans. Ant & Prop. Vol. 48, 1367-1375 (2000), by E. Bahar.

Duality Principle

- Idea: we have 2 sets of equations that are mathematically identical. Then, if you already have the solution to one set, you can easily get a solution to the 2nd (other) set by swapping "dual" quantities.

Dual Equations

$\boxed{\underline{J} \neq 0}$	$\boxed{\underline{M} \neq 0}$
$\nabla^2 \underline{A} + \beta^2 \underline{A} = -\mu \underline{J}$	$\nabla^2 \underline{F} + \beta^2 \underline{F} = -\epsilon \underline{M}$
$\underline{A}(\underline{r}) = \frac{\mu}{4\pi} \iiint_V \frac{\underline{J}(\underline{r}') e^{-j\beta R}}{R} d^3r'$	$\underline{F}(\underline{r}) = \frac{\epsilon}{4\pi} \iiint_V \frac{\underline{M}(\underline{r}') e^{-j\beta R}}{R} d^3r'$
$(\underline{F} = 0)$	$(\underline{A} = 0)$
$\nabla \times \underline{E}_A = -j\omega\mu \underline{H}_A$	$\nabla \times \underline{H}_F = +j\omega\epsilon \underline{E}_F$
$\nabla \times \underline{H}_A = \underline{J} + j\omega\epsilon \underline{E}_A$	$\nabla \times \underline{E}_F = -\underline{M} - j\omega\mu \underline{H}_F$
$(\underline{E}_F = \underline{H}_F = 0)$	$(\underline{E}_A =$
$\underline{H}_A = \frac{1}{\mu} \nabla \times \underline{A}$	$\underline{E}_F = -\frac{1}{\epsilon} \nabla \times \underline{F}$
$\underline{E}_A = -j\omega \underline{A} - \frac{j}{\omega\mu\epsilon} \nabla(\nabla \cdot \underline{A})$	$\underline{H}_F = -j\omega \underline{F} - \frac{j}{\omega\mu\epsilon} \nabla(\nabla \cdot \underline{F})$
$\underline{E} = \underline{E}_A$	$\underline{E} = \underline{E}_F$
$\underline{H} = \underline{H}_A$	$\underline{H} = \underline{H}_F$