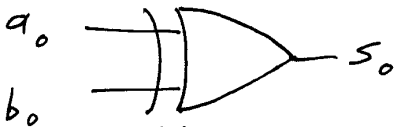
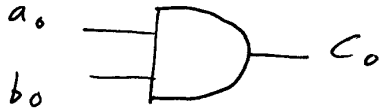
ADDING 2-BITS (HALF ADDER)

a_0	b_0	S_0	C_0
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



XOR



AND

SUM

$S_0 = a_0 \oplus b_0$

(EXCLUSIVE OR - a_0 OR b_0 BUT NOT BOTH)

$C_0 = a_0 \cdot b_0$

(AND: BOTH $a_0 = 1$ AND $b_0 = 1$ FOR $C_0 = 1$)

BUILDING-BLOCK LOGIC FUNCTIONS

AND

a	b	c
0	0	0
0	1	0
1	0	0
1	1	1

$c = a \cdot b$ LOGICAL PRODUCT

TRUTH TABLE

OR

a	b	c
0	0	0
0	1	1
1	0	1
1	1	1

$c = a + b$ LOGICAL SUM

NOT

a	$\bar{a}$
0	1
1	0

$\bar{a} = \text{NOT}(a)$

LOGICAL COMPLEMENT

REDUCING "XOR" TO THE BUILDING BLOCKS

a_0	b_0	F	g	s_0
0	0	0	0	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

$s_0 = F + g$

$\bar{a}_0$	b_0	$F = \bar{a}_0 \cdot b_0$
1	0	0
1	1	1
0	0	0
0	1	0

a_0	$\bar{b}_0$	$g = a_0 \cdot \bar{b}_0$
0	1	0
0	0	0
1	1	1
1	0	0

SUM-OF-PRODUCTS REALIZATION

$$s_0 = F + g = \bar{a}_0 \cdot b_0 + a_0 \bar{b}_0$$

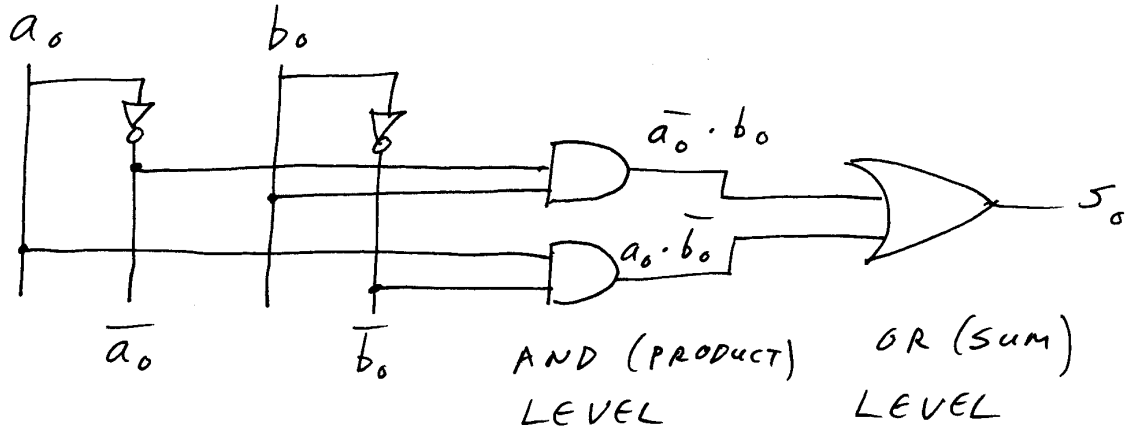
FROM TRUTH TABLE

	a_0	b_0	$\bar{a}_0 \cdot \bar{b}_0$	$\bar{a}_0 \cdot b_0$	$a_0 \cdot \bar{b}_0$	$a_0 \cdot b_0$	s_0
$\bar{a}_0\ \bar{b}_0$	0	0	1	0	0	0	0
$\bar{a}_0\ b_0$	0	1	0	1	0	0	1
$a_0\ \bar{b}_0$	1	0	0	0	1	0	1
$a_0\ b_0$	1	1	0	0	0	1	0
				✓	✓		

"MIN TERMS"

2-LEVEL SUM OF PRODUCTS

$$S_0 = \bar{a}_0 \cdot b_0 + a_0 \bar{b}_0$$



INSTEAD OF FOCUSING ON "1"s, LOOK AT "0"s

a_0	b_0	$S_0 = P \cdot q$	
0	0	0	← P SUPPLIES 0 HERE
0	1	1	
1	0	1	
1	1	0	← q SUPPLIES 0 HERE

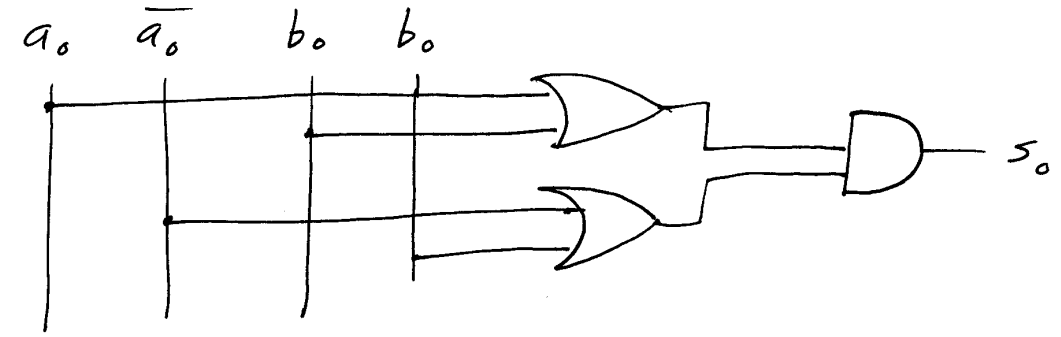
MAX TERMS

	$a_0 + b_0$	$a_0 + \bar{b}_0$	$\bar{a}_0 + b_0$	$\bar{a}_0 + \bar{b}_0$	
$a_0 + b_0$	0	1	1	1	✓
$a_0 + \bar{b}_0$	1	0	1	1	
$\bar{a}_0 + b_0$	1	1	0	1	
$\bar{a}_0 + \bar{b}_0$	1	1	1	0	✓

$$S_0 = (a_0 + b_0) \cdot (\bar{a}_0 + \bar{b}_0)$$

2-LEVEL PRODUCT OF SUMS

$$S_0 = (a_0 + b_0) \cdot (\bar{a}_0 + \bar{b}_0)$$



(INVERTERS NOT SHOWN)

DE MORGAN'S THEOREM

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$

PROOF

A	B	$\bar{A}$	$\bar{B}$	$A \cdot B$	$\overline{A \cdot B}$	$A + B$	$\overline{A + B}$	$\bar{A} \cdot \bar{B}$	$\bar{A} + \bar{B}$
0	0	1	1	0	1	0	1	1	1
0	1	1	0	0	1	1	0	0	1
1	0	0	1	0	1	1	0	0	1
1	1	0	0	1	0	1	0	0	0

MY FIRST JOB IN ACCOUNTING

DATA PROCESSING

IF (AGE < 60) AND (PAY < 2000)

INVITE_TO PICNIC ⇐ ELIMINATE PICNIC

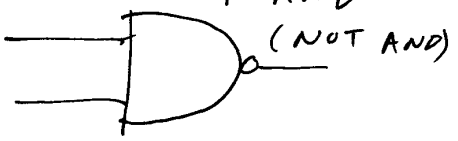
ELSE

LAYOFF

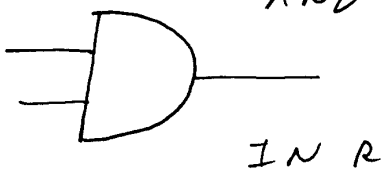
IF (AGE >= 60) OR (PAY ≥ 2000)

LAYOFF

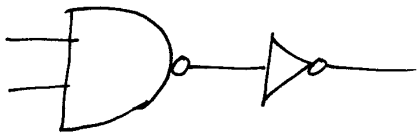
APPLICATION TO LOGIC CIRCUITS

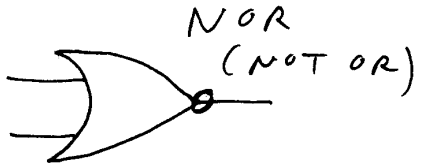


BASIC TTL GATE



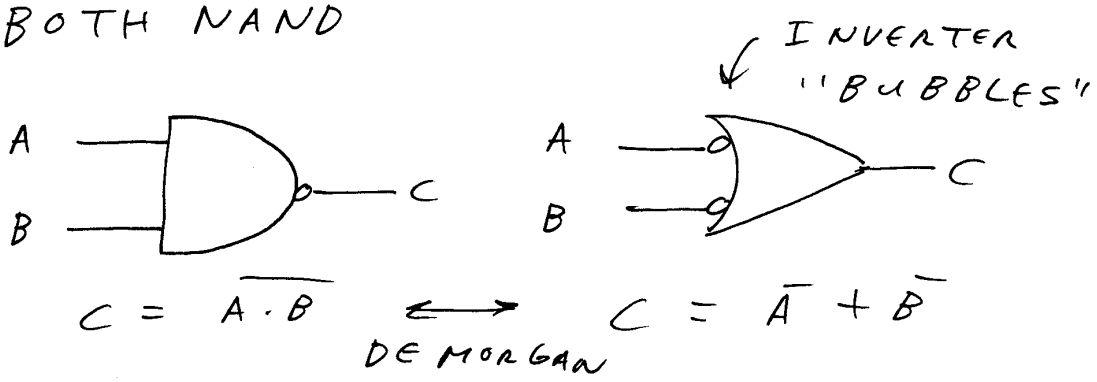
IN REALITY





BASIC CMOS GATE

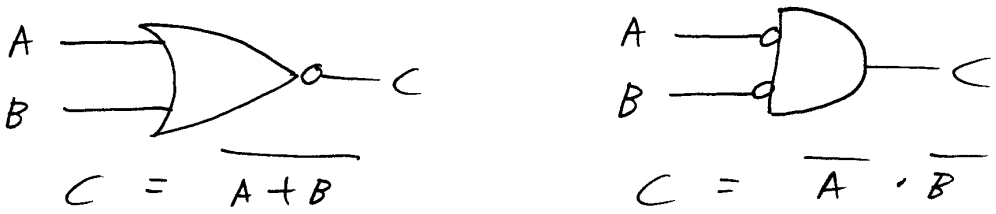
BOTH NAND



$C = \overline{A \cdot B} \longleftrightarrow C = \bar{A} + \bar{B}$

DE MORGAN

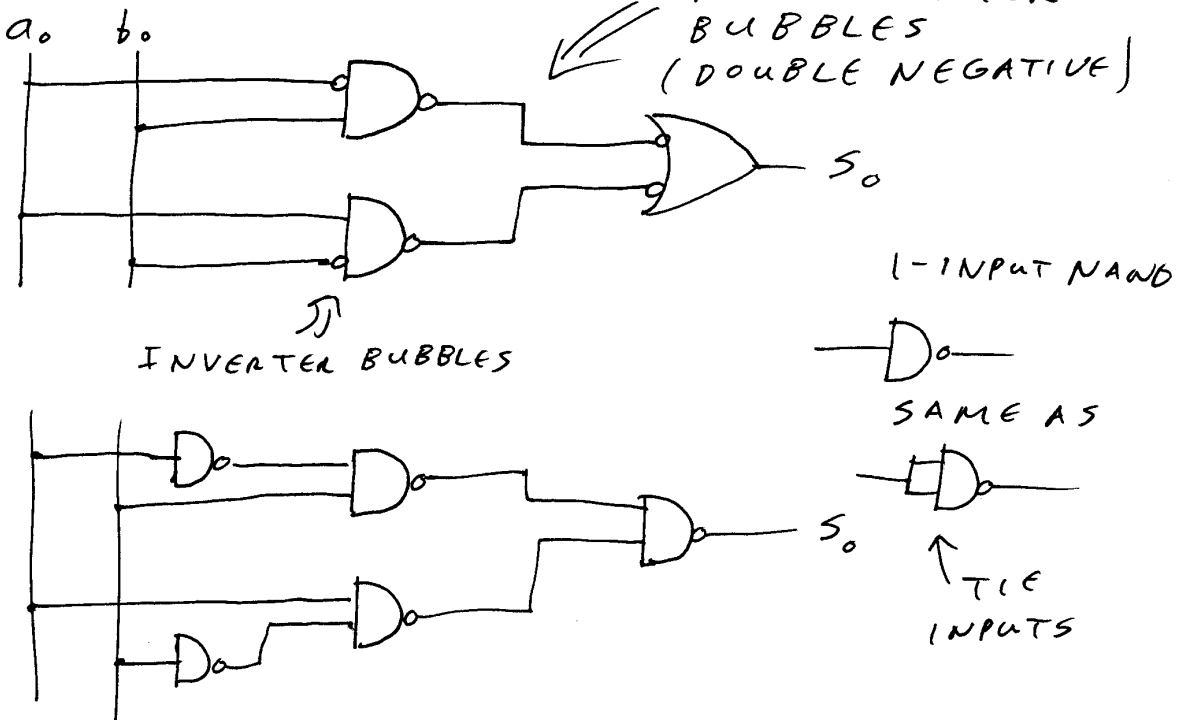
BOTH NOR



$C = \overline{A+B}$ $\qquad$ $C = \bar{A} \cdot \bar{B}$

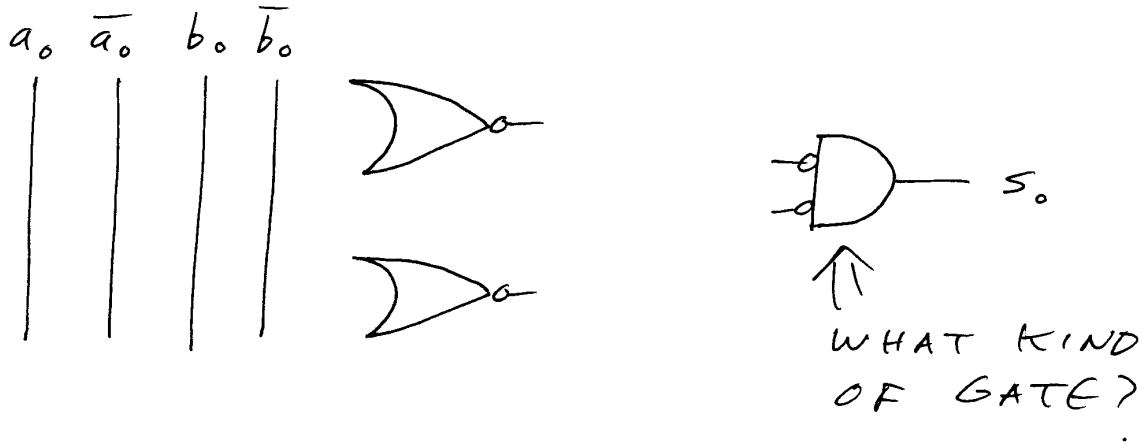
2-LEVEL SUM OF PRODUCTS

$S_0 = \bar{a}_0 b_0 + a_0 \bar{b}_0$



2-LEVEL PRODUCT OF SUMS

$$S_0 = (a_0 + b_0)(\bar{a}_0 + \bar{b}_0)$$



FULL ADDER

c_0	a_1	b_1	s_1	c_1
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

SOP $s_1 =$

POS $c_1 =$