AC POWER

$$p(t) = v(t) \cdot i(t)$$

ALWAYS!

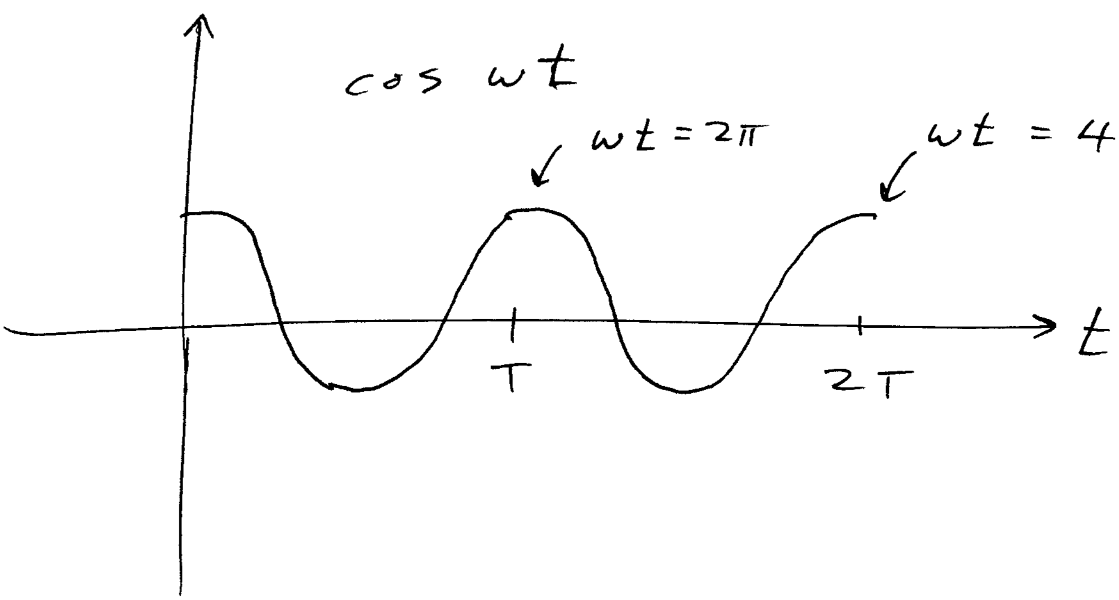
AVG POWER

$$P = \frac{1}{T} \int_0^T p(t)\,dt$$

$$= \frac{1}{T} \int_0^T v(t)\, i(t)\,dt$$

FOR $v(t) = \cos \omega t$,

$\omega T = 2\pi$ (ONE CYCLE) $\quad T = \frac{2\pi}{\omega}$



RESISTOR

$$v(t) = R\,i(t)$$

$$p(t) = v(t)\,i(t) = R\,i^2(t)$$

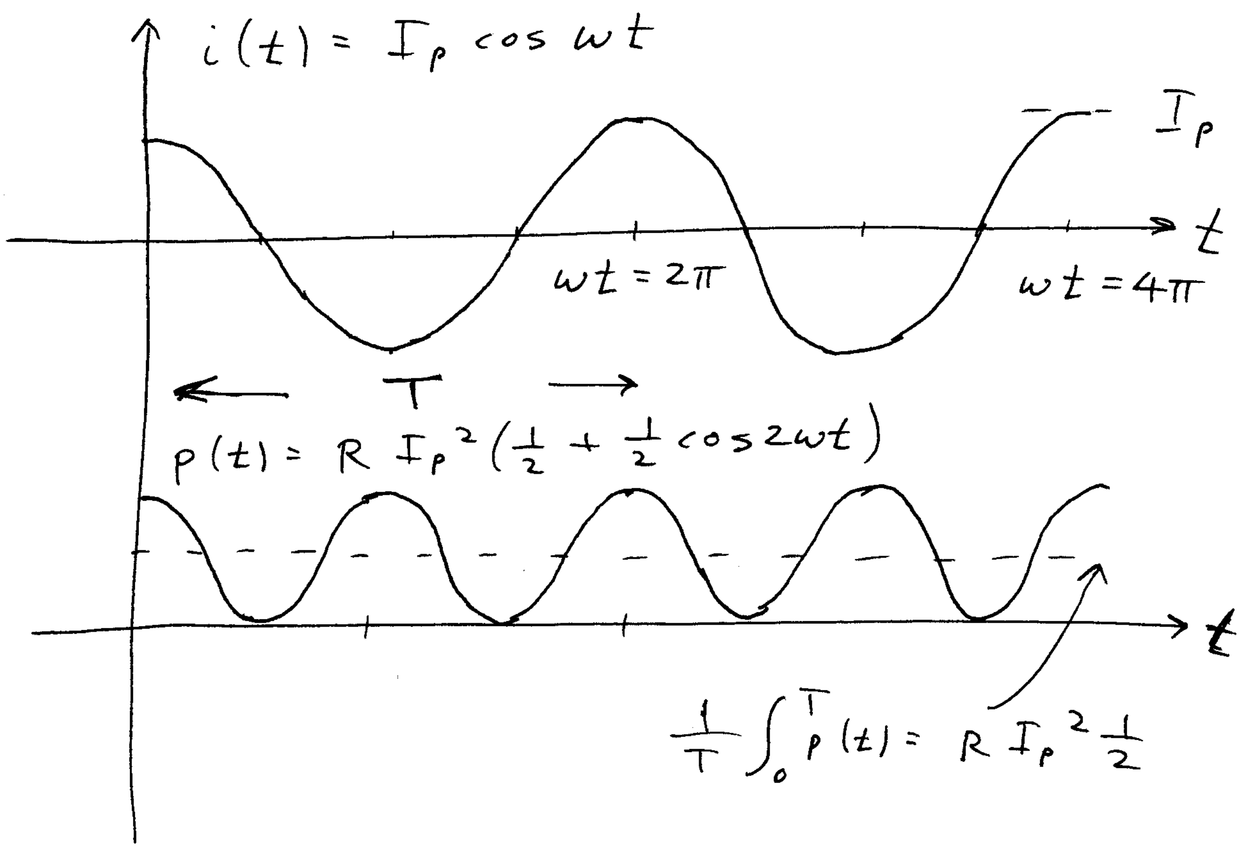
AVG POWER $\quad P = \frac{1}{T}\int_0^T R\,i^2(t)\,dt$

FOR $\; i(t) = I_p \cos \omega t$

$$i^2(t) = I_p^2 \cos^2 \omega t$$

$$= I_p^2\left(\underbrace{\tfrac{1}{2}}_{\text{AVG } \frac{1}{2}} + \underbrace{\tfrac{1}{2}\cos 2\omega t}_{\text{AVG } 0}\right)$$

$$P = \frac{1}{2} R\, I_p^2$$



THIS RMS BUSINESS

$$p(t) = R\, i^2(t)$$

$$\text{AVG}\{p(t)\} = \text{AVG}\{R\, i^2(t)\}$$

$$= R\, \text{AVG}\{i^2(t)\}$$

R JUST SCALES EVERYTHING

$$= R\left(\sqrt{\text{AVG}\{i^2(t)\}}\right)^2$$

$$= R\, I_{RMS}^2$$

WHERE "I_{EFF}" $= I_{RMS} = \sqrt{\text{AVG}\{i^2(t)\}}$

FOR $i(t) = I_p \cos \omega t$

$$\text{AVG}\{i^2(t)\} = \frac{1}{2} I_p^2$$

$$I_{RMS} = \sqrt{\frac{1}{2} I_p^2}$$

$$= \frac{1}{\sqrt{2}} I_p \quad !$$

WALL OUTLET: $V_{RMS} = 120\,V$

$$v(t) = \sqrt{2}(120)\cos\left(\frac{2\pi}{T} t\right)$$

$\hookrightarrow \omega = \frac{2\pi}{T}$

$\frac{1}{T} = 60\ Hz$

$$v(t) = 170 \cos(377t)$$

$\hookrightarrow 2\pi(60\ Hz)$

377 RAD/S

APPARENT POWER

$$|S| = |V_{RMS}||I_{RMS}| = \left|\frac{V}{\sqrt{2}}\right|\left|\frac{I}{\sqrt{2}}\right|$$

REASON FOR NOTATION GIVEN LATER

POWER FACTOR

$$p.F. = \frac{\text{AVG POWER}}{\text{APPARENT POWER}}$$

$$\text{RESISTOR } p.F. = \frac{R\,|I_{RMS}|^2}{R\,|I_{RMS}|\,|I_{RMS}|} = 1$$

INDUCTOR

$$v(t) = L\frac{di}{dt} \qquad i(t) = \underbrace{\sqrt{2}\,I_{RMS}}_{I_p}\cos\omega t$$

$$v(t) = \omega L\sqrt{2}\,I_{RMS}(-\sin\omega t)$$

$$p(t) = v \cdot i = 2\,\omega L\, I_{RMS}^2 \cos\omega t\,(-\sin\omega t)$$

$$= 2\,\omega L\, I_{RMS}^2\,\frac{1}{2}\,(-\sin 2\omega t)$$

$$= -Q\sin 2\omega t$$

$$Q = \omega L\, I_{RMS}^2$$

INDUCTOR POWER

$$P = AVG(-Q\sin 2\omega t) = 0!$$

AVG POWER

$$Q = \omega L I_{RMS}^2$$

REACTIVE POWER

$$p.F. = \frac{P}{|V_{RMS}||I_{RMS}|} = ?$$

$$|S| = |V_{RMS}||I_{RMS}| = \left|\frac{j\omega L I}{\sqrt{2}}\right|\left|\frac{I}{\sqrt{2}}\right|$$

APPARENT POWER

$$= \frac{\omega L}{2} I_p^2 = \omega L I_{RMS}^2 = Q!$$

$$p.F = \frac{P}{|S|} = \frac{0}{Q} = 0$$

CAPACITOR

$$v(t) = \frac{1}{C}\int i(t)\,dt \qquad i(t) = \sqrt{2} I_{RMS} \cos\omega t$$

$$v(t) = \frac{\sqrt{2}}{\omega C} I_{RMS} \sin\omega t$$

$$p(t) = v \cdot i = \frac{(\sqrt{2})(\sqrt{2})}{\omega C} I_{RMS} I_{RMS} \cos\omega t \sin\omega t$$

$$= \frac{2}{\omega C} I_{RMS}^2 \frac{1}{2} \sin 2\omega t$$

$$= -Q \sin 2\omega t$$

$$Q = -\frac{I_{RMS}^2}{\omega C}$$

CAPACITOR POWER

$$P = AVG\left(-Q \sin 2\omega t\right) = 0$$

$$Q = -\frac{I_{RMS}^2}{\omega C}$$

$$|S| = \frac{1}{\omega C}\left|I_{RMS}\right|^2 = |Q|$$

$$p.F. = \frac{P}{|S|} = 0!$$

MEANING OF Q - REACTIVE POWER

RESISTOR $P = |S| = |V_{RMS}|\,|I_{RMS}|$

GIVEN VOLTAGE, CURRENT DELIVERING AVG POWER

CAPACITOR
INDUCTOR
$P = 0$ ← NO AVG POWER

$$|S| = |Q| = |V_{RMS}|\,|I_{RMS}|$$

DRAWING CURRENT

$Q > 0$
(INDUCTOR - CURRENT LAGS VOLTAGE)

$Q < 0$
(CAPACITOR - CURRENT LEADS VOLTAGE)

RMS PHASOR $v(t) = V_p \cos \omega t + \theta_V$

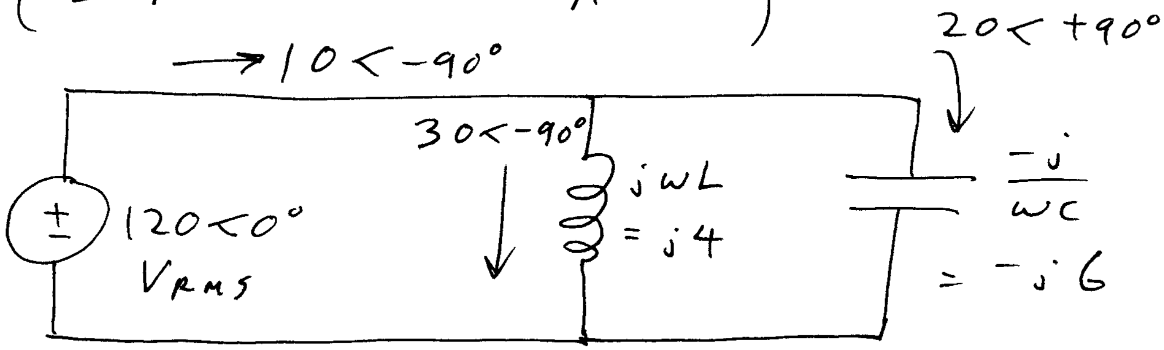
or

$$V_{RMS} = \frac{1}{\sqrt{2}} V \qquad |V_{RMS}| = \frac{1}{\sqrt{2}} V_p$$

PHASOR PHASOR

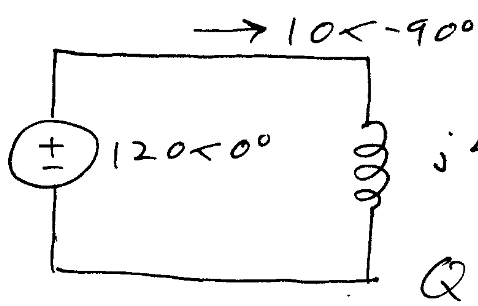
USE RMS PHASOR IN POWER PROBLEMS UNLESS OTHERWISE STATED

REACTIVE POWER ADDS
(SERIES OR PARALLEL)



$$Q_L = \omega L (30^2) = 4(900) = 3600 \qquad Q_{EFF} = 1200$$

$$Q_C = -\frac{(20)^2}{\omega C} = -6(400) = -2400$$



$$j4 \,||\, (-j6) = \frac{-j^2 24}{j4 - j6} = j12$$

$$Q = \omega L (10)^2 = (12)(100) = 1200$$