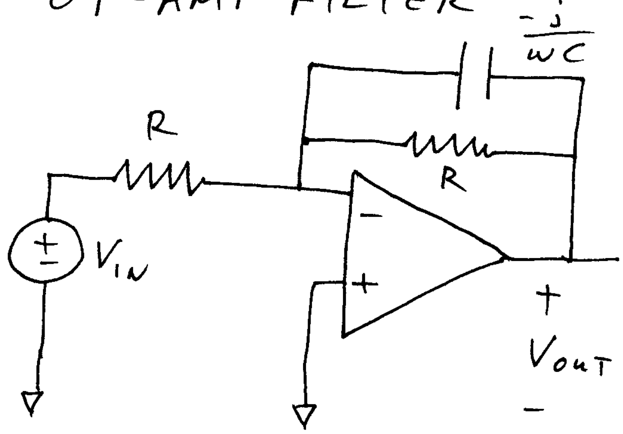
OP-AMP FILTER



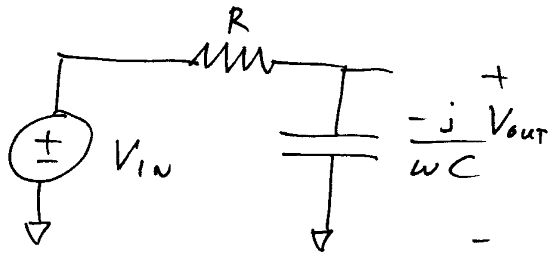
$$V_{OUT} = \frac{-Z_F}{Z_i} V_{IN}$$

$$Z_F = R \,||\, \frac{-j}{wC} = \frac{(R)\left(\frac{-j}{wC}\right)}{R - \frac{j}{wC}}$$

$$V_{OUT} = H V_{IN} \qquad -H = \frac{+Z_F}{Z_{IN}} = \frac{\frac{-j}{wC}}{R - \frac{j}{wC}} = \frac{1}{1 + jwRC}$$

$$-H = \frac{1}{1 + j\frac{w}{w_c}} \qquad w_c = \frac{1}{RC}$$

NON OP-AMP FILTER



PHASOR VOLTAGE DIVIDER

$$H = \frac{\frac{-j}{wC}}{R - \frac{j}{wC}} = \frac{1}{1 + jwRC}$$

FOR BOTH FILTERS

$$|H| = \frac{1}{\sqrt{1+\frac{\omega^2}{\omega_c^2}}} = \frac{1}{\sqrt{2}} \quad \text{WHEN } \omega = \omega_c$$

$$\approx 1 \quad \text{WHEN } \omega = .1\omega_c$$

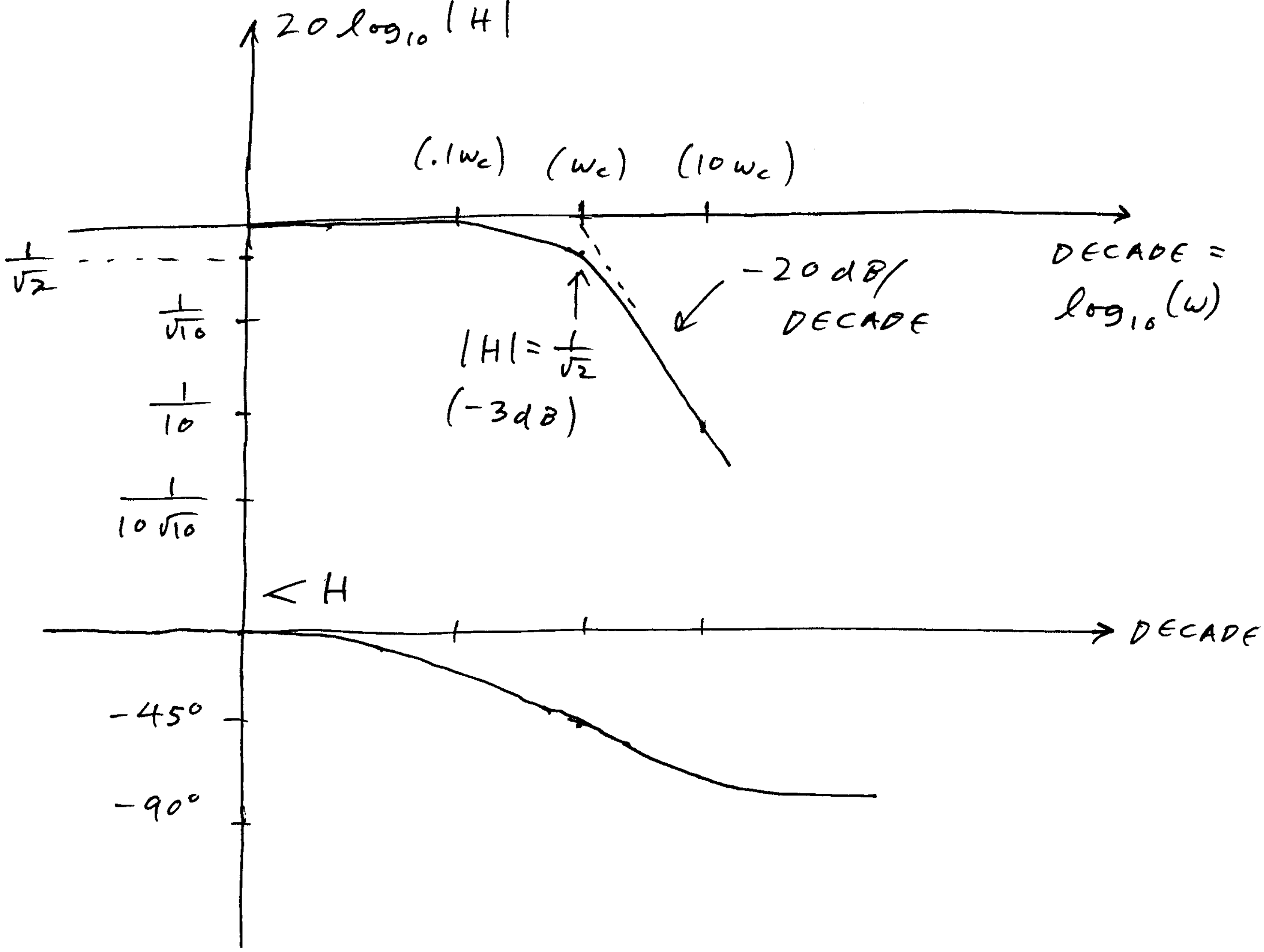
$$\approx .1 \quad \text{WHEN } \omega = 10\omega_c$$

$$\angle H = -\tan^{-1}\frac{\omega}{\omega_c} = -45^\circ \quad \omega = \omega_c$$

$$\approx 0^\circ \quad \omega = .1\omega_c$$

$$\approx -90^\circ \quad \omega = 10\omega_c$$

BODE PLOT dB VS DECADE



LET $R = 1000\,\Omega$

DESIGN FOR

$|H| = .9$ AT $\omega = 2000$ $\qquad \omega_c = ?$

$C = ?$

$$\frac{1}{\sqrt{1 + \frac{2000^2}{\omega_c^2}}} = .9$$

$$1 + \frac{2000^2}{\omega_c^2} = \frac{1}{(.9)^2}$$

$$(2000)^2 = \omega_c^2\left(\frac{1}{(.9)^2} - 1\right) \qquad \omega_c = 4129$$

$$\frac{1}{RC} = \omega_c \qquad C = \frac{1}{R\omega_c} = \frac{1}{(1000)(4129)}$$

$$= .242\,\mu F$$

DESIGN FOR $\angle H = -15^\circ$ AT $\omega = 400$

$\omega_c = ?$

MUST BE LEGAL

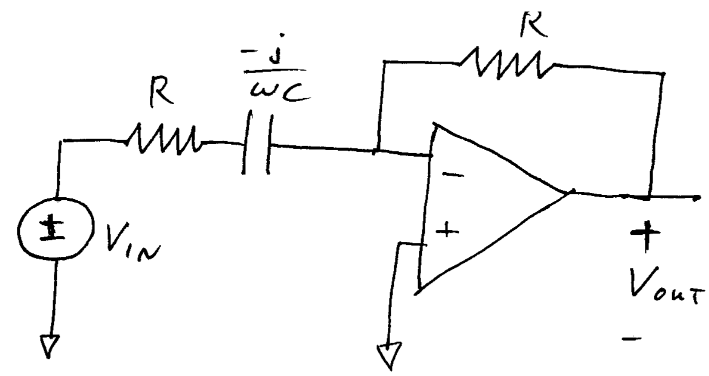
$0 \leq \angle H \leq -90^\circ$

$C = ?$

$$-15^\circ = -\tan^{-1}\frac{\omega}{\omega_c} \qquad \omega = \omega_c \tan 15^\circ$$

$$\omega_c = \frac{\omega}{\tan 15^\circ} = 1492$$

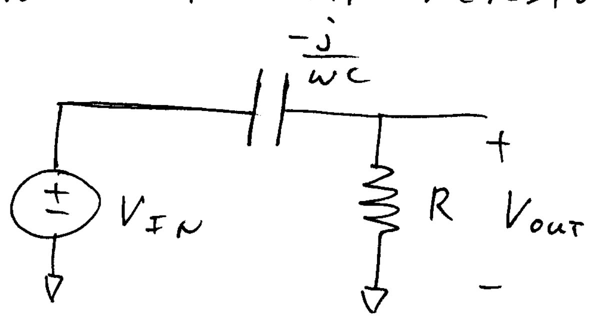
$$C = \frac{1}{R\omega_c} = \frac{1}{(1000)(1492)} = .67\,\mu F$$



$$-H = -\frac{V_{OUT}}{V_{IN}} = \frac{Z_F}{Z_I} = \frac{R}{R - \frac{j}{\omega C}}$$

$$= \frac{1}{1 + \frac{\omega_c}{j\omega}} \qquad \omega_c = \frac{1}{RC}$$

NON OP-AMP VERSION



$$H = \frac{R}{R - \frac{j}{\omega C}}$$

$$= \frac{1}{1 + \frac{\omega_c}{j\omega}}$$

$$\omega_c = \frac{1}{RC}$$

$$|H| = \frac{1}{\sqrt{1 + \frac{\omega_c^2}{\omega^2}}} \qquad \angle H = +\tan^{-1}\frac{\omega_c}{\omega}$$

FOR BOTH FILTERS

$$|H| = \frac{1}{\sqrt{1+\frac{\omega_c^2}{\omega^2}}} = \frac{1}{\sqrt{2}} \qquad \omega = \omega_c$$

$$\approx .1 \qquad \omega = .1\,\omega_c$$

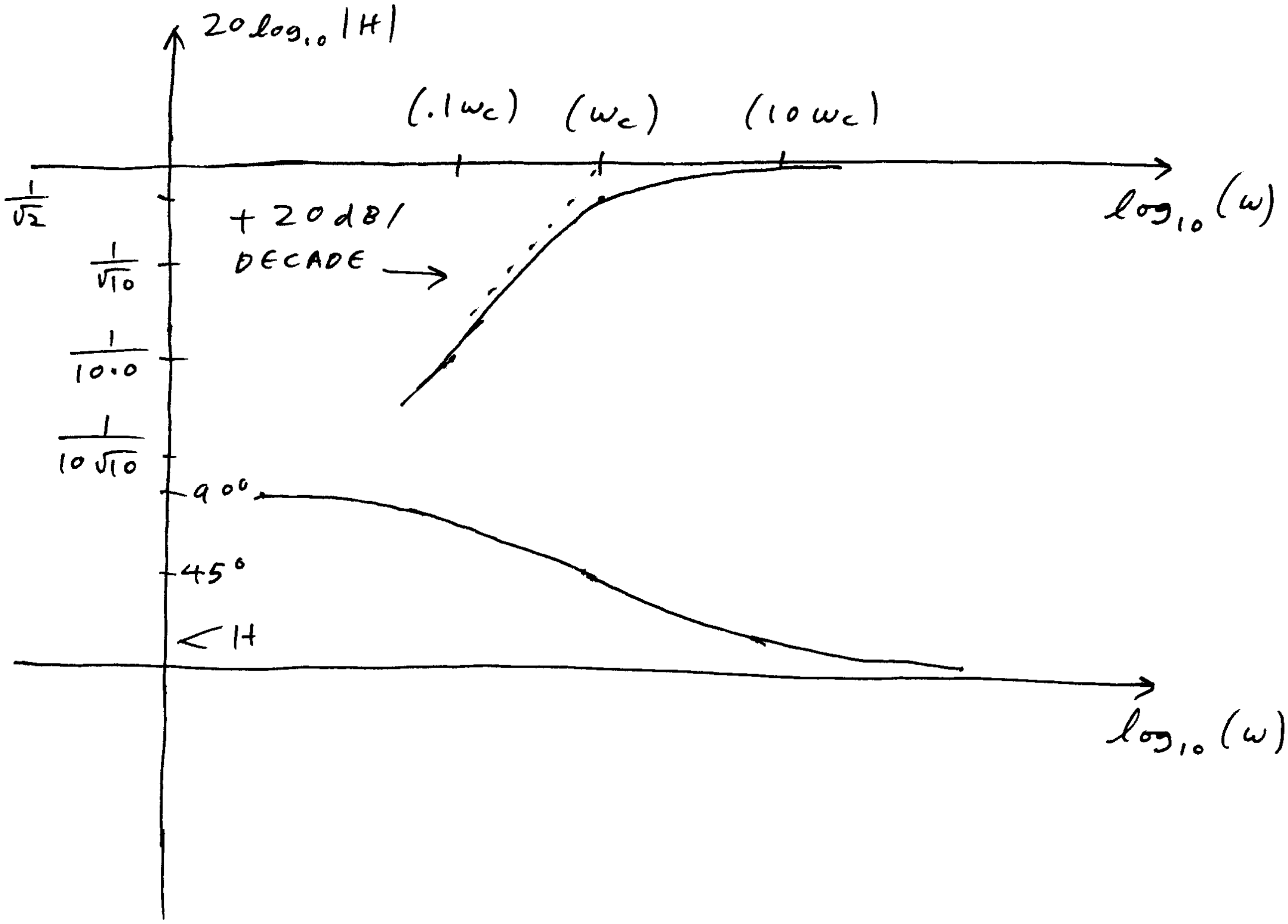
$$\approx 1 \qquad \omega = 10\,\omega_c$$

$$\angle H = \tan \frac{\omega_c}{\omega} = +45^\circ \qquad \omega = \omega_c$$

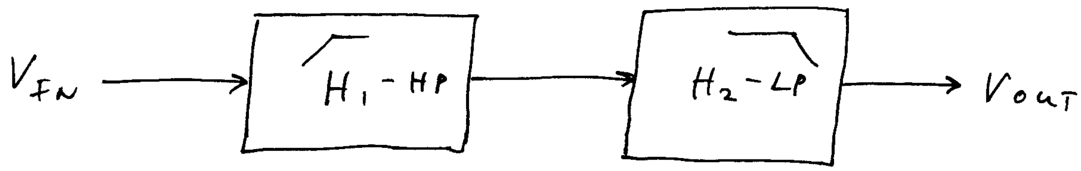
$$\approx +90^\circ \qquad \omega \to 0 \quad \frac{\omega_c}{\omega} \to \infty$$

$$\approx 0^\circ \qquad \omega \to \infty \quad \frac{\omega_c}{\omega} \to 0$$

BODE PLOT

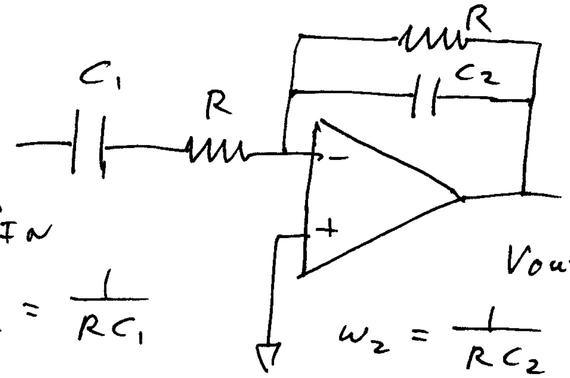


COMBINE FILTERS



$$H = H_1(\omega) \cdot H_2(\omega)$$

$$\log_{10} H = \log_{10} H_1 + \log_{10} H_2$$



$$\omega_1 = \frac{1}{RC_1} \qquad \omega_2 = \frac{1}{RC_2}$$

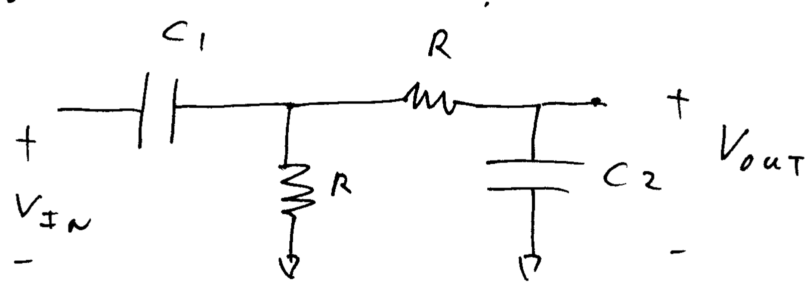
$$\omega_1 < \omega_2$$

$$H = \left(\frac{1}{1 + \frac{\omega_1}{j\omega}}\right)\left(\frac{1}{1 + j\frac{\omega}{\omega_2}}\right)$$

$$|H| = \frac{1}{\sqrt{1 + \frac{\omega_1^2}{\omega^2}}\sqrt{1 + \frac{\omega^2}{\omega_2^2}}}$$

$$\angle H = \tan^{-1}\frac{\omega_1}{\omega} + -\tan^{-1}\frac{\omega}{\omega_2}$$

DOES THIS WORK?



COMBINED FILTER $\quad \omega_2 \approx 10\,\omega_1$

$$|H| = \frac{1}{\sqrt{1 + \frac{\omega_1^2}{\omega^2}}} \, \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_2^2}}}$$

$$\angle H = \tan^{-1}\frac{\omega_1}{\omega} - \tan^{-1}\frac{\omega}{\omega_2}$$

	$\lvert H \rvert$	$\angle H$
$.1\,\omega_1$	$(.1)(1)$	$+90^\circ$
ω_1	$(\frac{1}{\sqrt{2}})(1)$	45°
ω_2	$(1)(\frac{1}{\sqrt{2}})$	-45°
$10\,\omega_2$	$(1)(.1)$	-90°

BODE PLOT