COSINES, SINES, COMPLEX NUMBERS, AC CIRCUITS IN A NUTSHELL

AC VOLTAGE AND CURRENT

$v_1(t) = 20 \cos 50t$

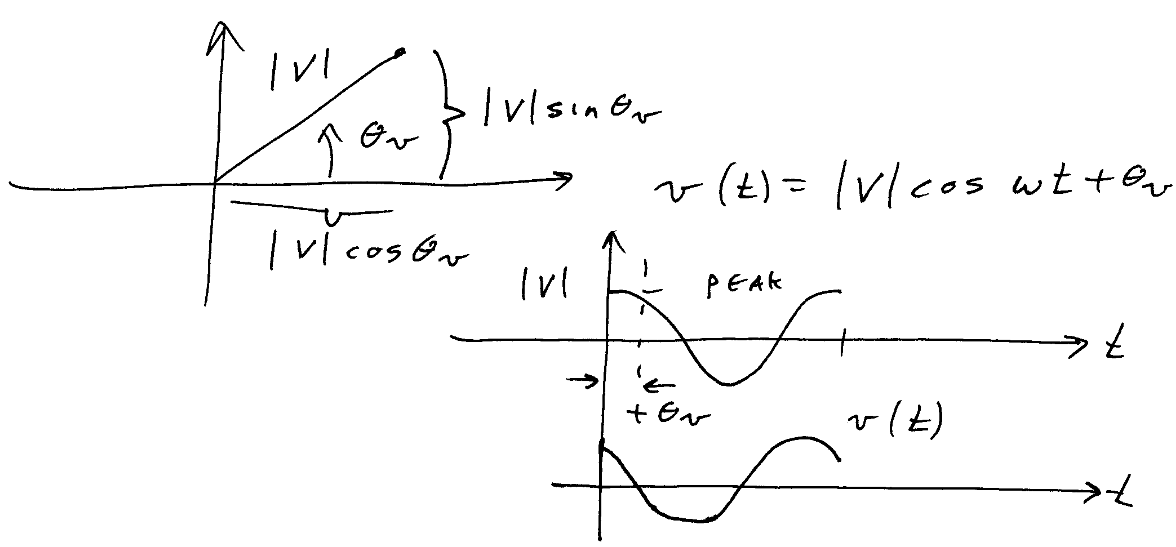
$v_2(t) = 12 \cos 40t + 30°$

$v_3(t) = 8 \cos 2\pi 60t - 90°$
$= 8 \sin 2\pi 60t$

PHASOR - REPRESENT COSINE WAVE AS COMPLEX NUMBER

$V = |V| \angle \theta_v = |V| e^{j\theta_v}$ POLAR

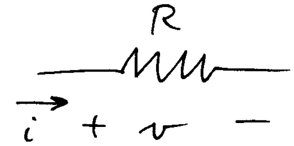
$= |V| \cos \theta_v + j |V| \sin \theta_v$



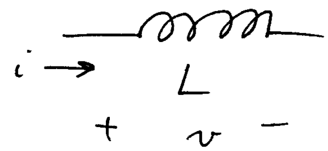
WHEN DO WE MULTIPLY PHASORS?

WHEN WE DO

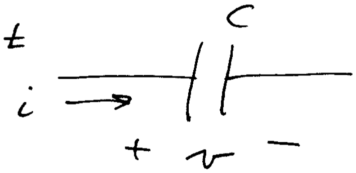
$$v(t) = R\,i(t)$$



$$v(t) = L\frac{d\,i(t)}{dt}$$



$$v(t) = \frac{1}{C}\int i(t)\,dt$$



RESISTOR	WAVEFORM	PHASOR

RESISTOR

$$v(t) = R\,i(t)$$

FOR $i(t) = \cos\omega t$

$$v(t) = R\cos\omega t$$

$$V = R\,I = R\cdot 1\angle 0^\circ = R\angle 0^\circ$$

INDUCTOR

$$v(t) = L\frac{di(t)}{dt} = -\omega L\sin\omega t = \omega L\cos(\omega t + 90^\circ)$$

$$V = j\omega L\,I = \omega L\angle 90^\circ\; 1\angle 0^\circ = \omega L\angle 90^\circ$$

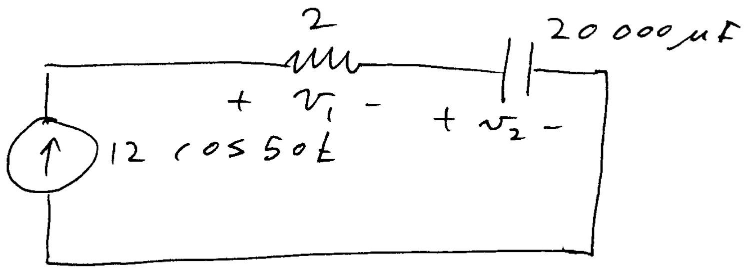
CAPACITOR

$$v(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{\omega C}\sin\omega t = \frac{1}{\omega C}\cos(\omega t - 90^\circ)$$

$$V = \frac{1}{j\omega C}\,I = \left(\frac{-j}{\omega C}\right)(1) = \frac{1}{\omega C}\angle -90^\circ\; 1\angle 0^\circ = \frac{1}{\omega C}\angle -90^\circ$$

WHEN DO WE ADD PHASORS?

WHEN WE DO KVL, KCL TO ADD VOLTAGE OR CURRENT WAVEFORMS



$$V_1 = 2 I_S = (2)(12) = 24 \angle 0^\circ$$

$$v_1(t) = 24 \cos 50t$$

$$V_2 = \frac{1}{j\omega C} I_S = \frac{1}{j\,50\,(20\,000)10^{-6}} I_S$$

$$= (1 \angle -90^\circ)(12) = 12 \angle -90^\circ$$

$$v_2(t) = 12 \cos 50t - 90^\circ = 12 \sin 50t$$

LISTEN TO ME NOW & BELIEVE ME LATER

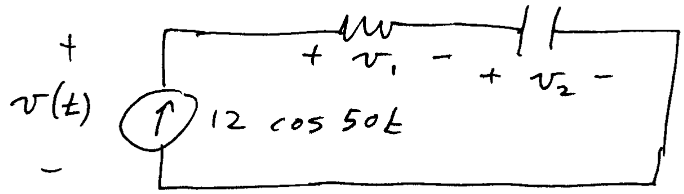
$$v_2(t) = \frac{1}{C} \int 12 \cos 50t \, dt$$

$$= \frac{1}{50C} 12 \sin 50t$$

$$= \frac{1}{50\,(20\,000)10^{-6}} 12 \sin 50t$$

$$= 12 \sin 50t$$

BUT PROFESSOR, HOW DO WE ADD PHASORS (AND WAVEFORMS)?



$v(t) = v_1(t) + v_2(t)$

$v(t) = 24\cos 50t + 12\sin 50t = ?$

$V = 24\angle 0^\circ + 12\angle -90^\circ = ?$

$= 24 + j0 + 0 - j12$

$= 24 - j12$

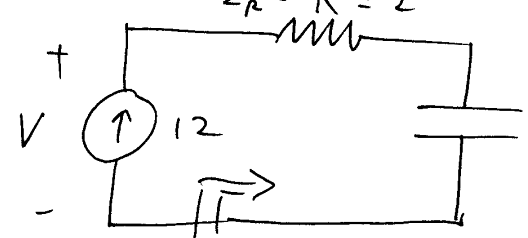
$v(t) = 24\cos 50t + 12\sin 50t$ NO GAIN

$V = 26.8\angle -26.6^\circ$

$v(t) = 26.8\cos 50t - 26.6^\circ$

IMPEDANCE $Z_R = R$ $Z_C = \frac{1}{j\omega C}$ $Z_L = j\omega L$

$Z_R = R = 2$



$Z_C = \frac{1}{j\omega C} = \frac{1}{j} = -j$

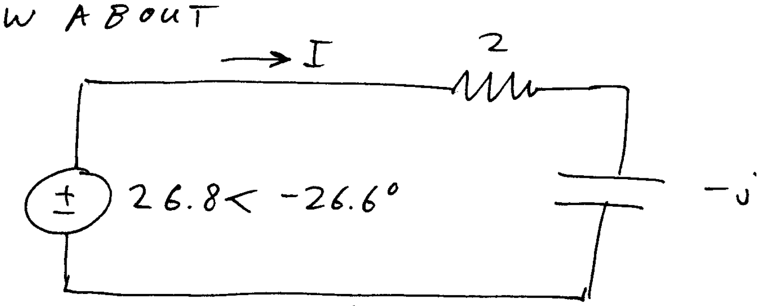
$Z_{EFF} = 2 + (-j)$

$V = (Z_{EFF})(I) = (2-j)(12)$

$= 24 - j12 = 26.8\angle -26.6^\circ$

$v(t) = 26.8\cos 50t - 26.6^\circ$

HOW ABOUT

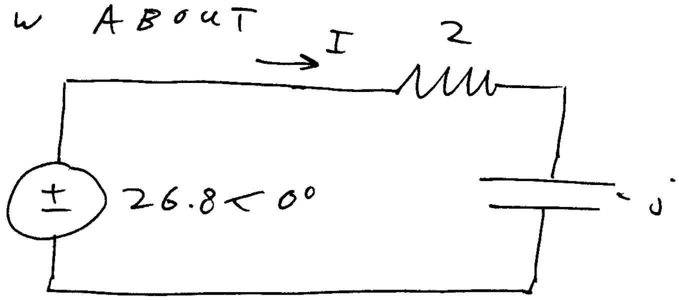


$$I = \frac{V}{Z_{EFF}} = \frac{V}{2-j}$$

$$= \frac{26.8\angle -26.6^\circ}{2.236\angle -26.6^\circ} = 12\angle 0^\circ$$

$$i(t) = 12\cos 50t$$

HOW ABOUT



$$I = \frac{26.8\angle 0^\circ}{2-j} = \frac{26.8\angle 0^\circ}{2.236\angle -26.6^\circ}$$

$$= 12\angle +26.6^\circ$$

$$i(t) = 12\cos 50t + 26.6^\circ$$

WHAT IF $V = 53.6\angle 0^\circ$?

$i(t) =$ __________