## Development of the Stator Referred Induction Machine Model

The development of the stator referred model, following the method used in developing the conventional transformer equivalent circuit, offers a useful perspective on model development. This approach follows the scheme leading to the concept of the ideal transformer by neglecting all parameters not essential to the basic behavior. For the induction machine the list of initially neglected quantities includes:

1) all leakage fluxes
2) the stator resistance
3) the magnetizing current
4) the core loss

The rotor resistance must be retained for, as is well known , the torque is always zero if the rotor resistance is zero.

With the leakage fluxes neglected, the rotor, stator and air gap fluxes are identical. When the stator resistance is also neglected, the amplitude of this flux is given by Faraday's Law.

$$V_s = 0.707\ \omega_e N_s \phi = 4.44\ f_e N_s \phi \qquad (1)$$

On the rotor side Faraday's Law can be used to find the rotor induced voltage by using the fact that the rotor frequency differs from the stator frequency because of the rotor motion. The rotor frequency can be expressed as $Sf_e$ resulting in

$$V_r = 4.44\ (Sf_e) N_r \phi \quad \text{(a)} \qquad \text{where } S = \frac{RPM_{synch} - RPM_{rotor}}{RPM_{synch}} \quad \text{(b)} \qquad (2)$$

which differs from the stator voltage because of

1) Turns $N_r$ instead of $N_s$

2) Frequency $Sf_e$ instead of $f_e$

From (1) and (2) the relation between the stator and rotor voltages is

$$V_r = S\ (N_r/N_s)\ V_s \qquad (3)$$

And, neglecting magnetizing current, Ampere's Law results in

$$N_s I_s = N_r I_r \qquad (4)$$

The circuit model shown in Fig.1, using two dependent generators, can be used to model these two equations ((3) and (4))

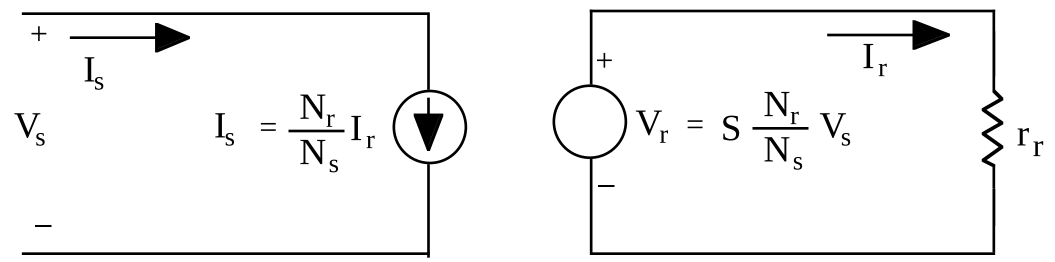


**Fig 1 Induction Machine Idealized Model using Two Dependent Generators**

The equation for the rotor current is

$$I_r = \frac{S\frac{N_r}{N_s} V_s}{r_r} = \frac{\text{Variable voltage (and frequency)}}{\text{Constant Resistance}} \tag{5}$$

Rewriting the equation by dividing top and bottom by S

$$I_r = \frac{\frac{N_r}{N_s} V_s}{\frac{r_r}{S}} = \frac{\text{Constant voltage (and frequency)}}{\text{Variable Resistance}} \tag{6}$$

Using this result the circuit of Fig.1, which has a varying secondary voltage, can be redrawn as shown in Fig. 2 in which the secondary voltage is constant.

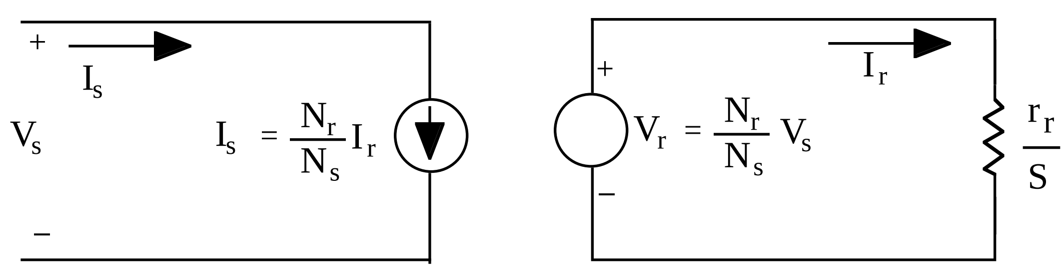


**Fig 2 Induction Machine Idealized Model using Two Dependent Generators and Variable Rotor Resistance**

Fig.2 can be simplified by observing that an ideal transformer of ratio $N_s/N_r$ can be used to replace the two dependent generators yielding the circuit of Fig. 3.

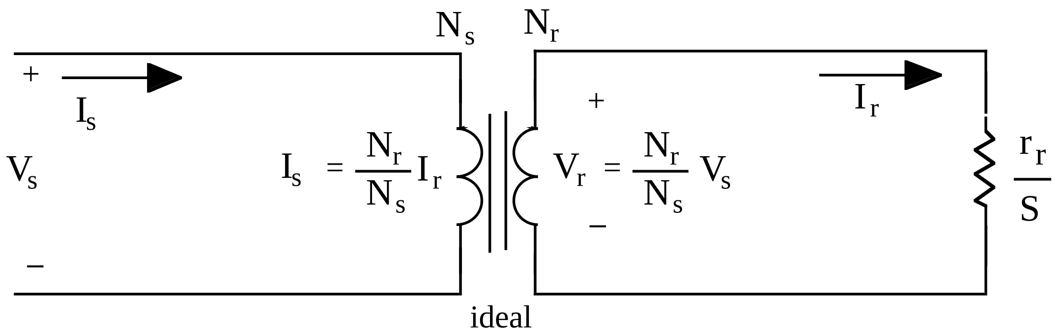


**Fig 3 Induction Machine Idealized Model using Ideal Transformer and Variable Rotor Resistance**

A stator referred model can then be obtained by simply transferring all rotor side quantities to the stator resulting in the model shown in Fig,.4.

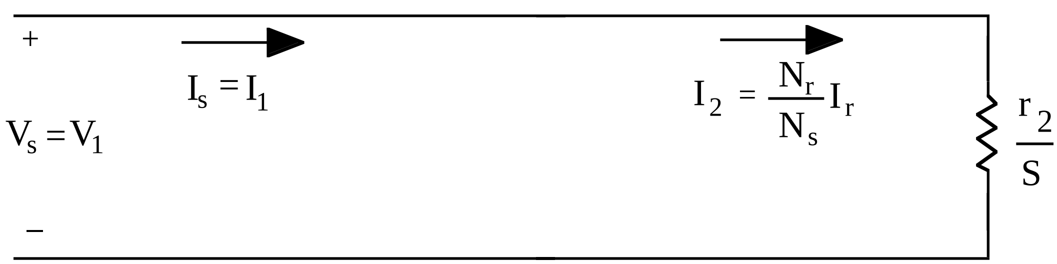


**Fig 4 Induction Machine Idealized Model with all Quantities Referred to the Stator**

where:

$$r_2 = \frac{N_s^2}{N_r^2} r_r \qquad I_2 = \frac{N_r}{N_s} I_r \qquad I_1 = I_s \qquad V_1 = V_s \qquad (7)$$

Power and torque computations can be clarified by redrawing the circuit with the referred rotor resistance r2 shown explicitly. This requires subtracting $r_2$ from the resistor $r_2/S$ to yield the new variable $(r_2/S)(1–S)$ as shown in Fig.5.

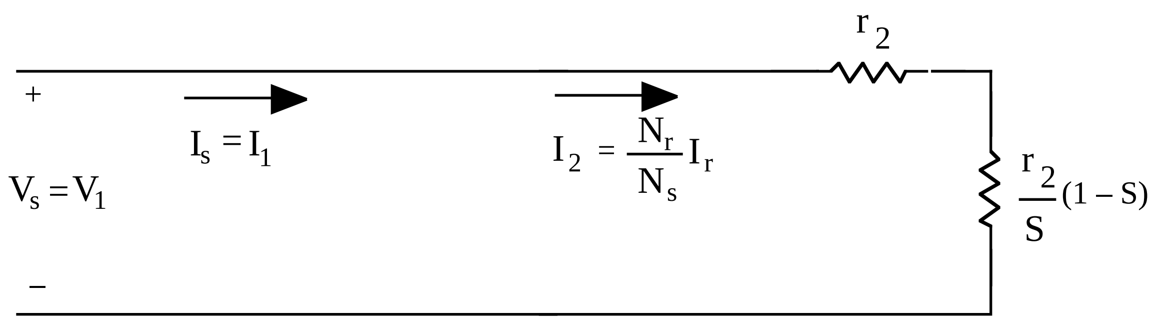


**Fig 5 Induction Machine Idealized Model with $r_2$ Explicitly Shown**

and the power and torque are given by the expressions

$$P_{in} \approx 3\, V_1\, I_1 \cos\theta \approx 3\, I_2^2 \frac{r_2}{S} \quad (a) \qquad P_{out} \approx P_{in} - 3\, I_2^2\, r_2 \approx 3\, I_2^2 \frac{r_2(1-S)}{S} \quad (b) \qquad (8)$$

$$T = \frac{P_{out}}{\omega_{rm}} \approx \frac{3\, I_2^2 \frac{r_2(1-S)}{S}}{\omega_{em}\,(1-S)} \approx \frac{3\, I_2^2 \frac{r_2}{S}}{\omega_{em}} \qquad (9)$$

Solving for the current using the model of Fig 4 or 5 yields the approximate expressions for the slip dependence of the input current and output torque.

$$I_1 \approx I_2 \approx \frac{V_1}{r_2} S \quad (a) \qquad T \approx \frac{3\, I_2^2 \frac{r_2}{S}}{\omega_{em}} \approx \frac{V_1^2}{\omega_{em}\, r_2} S \quad (b) \qquad (10)$$

The linear torque vs. slip characteristic predicted by Eq. 10b and illustrated in Fig 6 is an excellent approximation for small slip.  The stator current approximation is not as good because of the

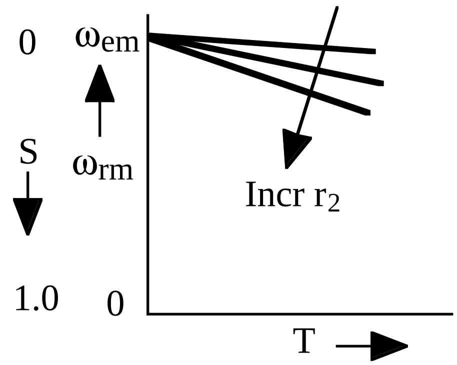


**Fig 6 Induction Machine Torque - Slip Curves for Small Slip**

magnetizing current which is much larger than in a transformer because of the required air gap. Adding the magnetizing reactance results in the circuit of Fig 7 which is a reasonable model for stator current behavior for small slip.

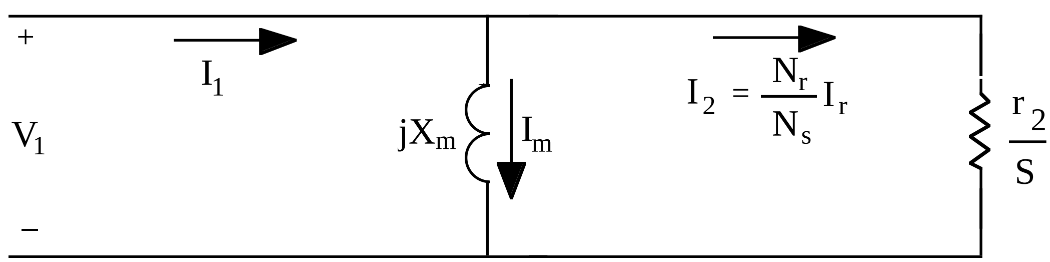


**Fig 7 Induction Machine Idealized Model with Magnetizing Branch Added**

While the model of Fig.7 clearly represents the nature of induction machine behavior for the small slip, normal operating region, it fails to predict the important characteristics of large slip operation. In particular it does not predict the existance of a peak value in the torque vs. slip characteristic and also fails to correctly represent the dependence of the starting torque (slip = zero) on motor parameters. Both of these properties are strongly dependent on the leakage reactance and the addition of these elements to the circuit of Fig.7, following the lead of the transformer model, yields the large slip model shown in Fig.8.

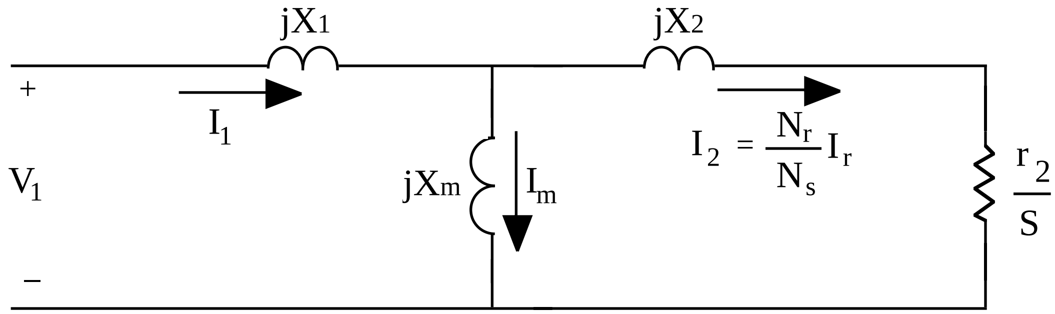


**Fig 8 Induction Machine Idealized Model with Leakage Reactance Added (Large Slip Model)**

Torque computations are often done by using Thevenin's Theorem to replace the circuit to the left of $r_2/S$ by $V_{th}$ and $X_{th}$. Quite acceptable approximate results for both peak torque and starting torque can be obtained by neglecting the magnetizing reactance $X_m$ (since typically $X_m >> X_1$).

$$V_{th} = V_1 \frac{X_m}{X_m + X_1} \approx V_1 \tag{11}$$

$$X_{th} = X_2 + \frac{X_1 \text{ x } X_m}{X_1 + X_m} \approx X_1 + X_2 \tag{12}$$

The last two results are equivalent to removing $X_m$ from the circuit of Fig 8 which demonstrates the dominance of the rotor resistance $r_2$ and the total leakage reactance $X_1 + X_2$ in controlling the large slip behavior of the induction machine.

Adding the stator resistance and the core loss resistor yields the complete stator referred equivalent circuit as shown in Fig 9. In most cases one or more of the parameters can be neglected without seriously affecting results. Probably the most demanding calculations are losses and efficiency for which all parameters are needed.

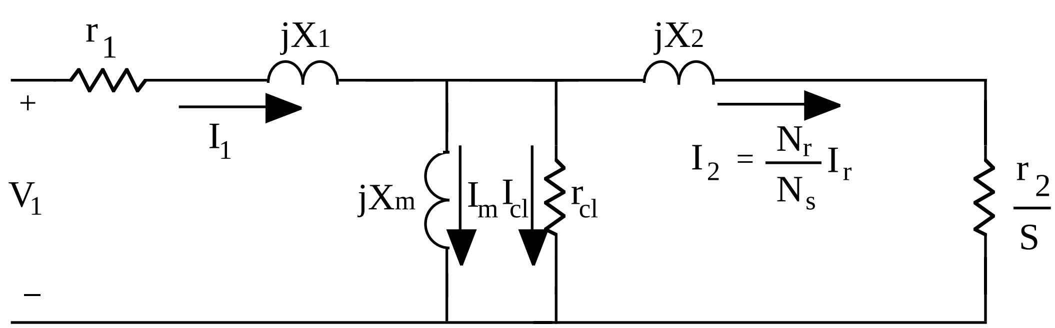


**Fig 9 Conventional Stator Referred Induction Machine Circuit Model**