Magnetics for Power Devices

1) Fundamental Laws of Magnetic Systems

The three fundamental laws that govern the design and operation of magnetic systems are:

1) Ampere's Law - The closed line integral of the magnetic intensity H around a closed path C is equal to the current enclosed by the path C.

$$\int_C H \bullet dL = \text{current enclosed by C} \qquad (1)$$

In most power devices the value of H is constant over well defined regions and the integral can be replaced by a sum. The enclosed current usually results from a conductor linking the contour C, N times, resulting in the following form of Ampere's Law

$$\boldsymbol{\sum H_k L_k = NI = \text{magnetizing mmf}} \qquad (2)$$

In this form Ampere's Law is similar to Kirchoff's voltage law with the

magnetizing mmf NI acting as the source of the magnetic field
and
the HL products as the load quantities being magnetized.

This is an important conceptual picture of the creation of magnetic fields in power devices

2) **Gauss' Law** - The net magnetic flux through any closed surface is zero.

$$\int_S B \bullet dA = 0 \qquad (3)$$

Gauss' Law simply states that magnetic flux is conserved; there are no beginning or ending points of magnetic flux. This is analogous to Kirchoff's current law and in the sense of a well defined magnetic circuit can be stated as

the incoming and outgoing magnetic fluxes at any branch point are equal

3) **Faraday's Law** - The magnetically induced voltage around any path is equal to the time rate of change of the magnetic flux linking the path.

$$v = \frac{d\lambda}{dt} \tag{4}$$

Usually the flux linkage is created by the same flux passing through a collection of N turns resulting in

$$\lambda = N\,\phi \qquad\qquad \mathbf{v = N\,\frac{d\phi}{dt}} \tag{5}$$

A very important special case of Faraday's Law exists when the magnetic flux varies sinusoidally

$$\phi = \phi_m \sin \omega t \qquad\qquad v = N\,\frac{d(\phi_m \sin \omega t)}{dt} = N\,\omega\,\phi_m \cos \omega t \tag{6}$$

from which the relation between the rms induced voltage V and the flux is

$$\mathbf{V} = \frac{N\,\omega\,\phi_m}{\sqrt{2}} = \frac{2\pi}{\sqrt{2}}\,f\,N\phi_m \;\mathbf{=\;4.44\;\;f\,N\phi_m} \tag{7}$$

In AC magnetic systems this expression plays a dominant role in establishing the flux level in the system, since usually the IR drop is negligibly small. The applied voltage, frequency and turns then determine the flux level and the magnetizing mmf (current) must adjust to give this flux.

In the AC case the magnetizing mmf is still the source of the flux
but the level is set by Faraday's Law

2) Magnetic Materials

The relation between the flux density B and the field intensity H depends on the material in which the field is established. There are two important cases:

1) Non-magnetic (sometimes called linear) materials - in which the relation between B and H is the linear relation

$$B = \mu_0 \mu_R H \tag{8}$$

where μ_R is the relative permeability and μ_0 is the permeability of free space (vacuum)

$$\boldsymbol{\mu_0 = 4\pi \times 10^{-7}} \quad \textbf{Tesla/(Amp/meter) or Henry/meter} \tag{9}$$

For most materials the relative permeability is very close to one and for engineering work is usually taken as one. The most common engineering material in this class is air but the non-ferrous metals and various plastics and other electrical insulating materials also find use.

2) Ferromagnetic materials - have an internal magnetization that can aid or oppose the applied (external) magnetization resulting in the following properties:

a) a very high but also quite variable relative permeability depending on the value of B.

b) a tendency to confine the total magnetic flux within the material itself, thus acting somewhat as a "flux conductor" similar to the wires in an electric circuit.

c) a high but finite *saturation flux density* above which the material behaves as if it were non-magnetic.

d) a tendency for the material to oppose a change in B. This property is referred to as *hysteresis* and creates a situation where B depends not only on the present value of H but also on the past history of H.

e) the ability to retain magnetization even after the external magnetization is removed. This is a special case of hysteresis which is termed permanent magnetism.

f) with AC magnetization there are irreversible heat losses that result from eddy currents in the material, since it is electrically conducting, and from internal losses related to hysteresis.
The *eddy current loss* and *hysteresis loss* together are the *core loss*.

3) Application of ferromagnetic materials in power devices - in most magnetic field power devices the desired result is a specific level of flux over a specified region with a reasonable (small) value of magnetizing mmf. To meet these requirements, ferromagnetic material:

a) properties "a" and "b" (high permeability and flux confinement) are ideal, allowing low H and hence low magnetizing mmf and helping confine the flux to the desired region. Both properties are of enormous practical significance, the high permeability greatly reduces the amount of magnetizing current and the flux confinement (or flux guiding) is invaluable in placing magnetic flux where it is desired and preventing flux "leakage".

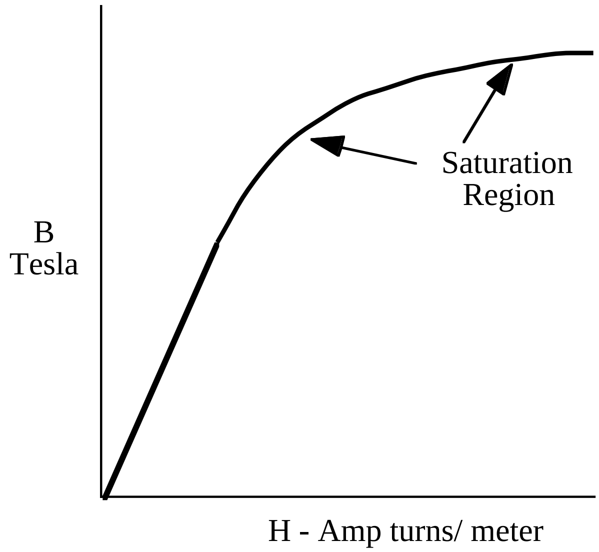
b) property "c" (saturation) imposes a limitation which requires that the design "avoid saturation" to retain the advantages of low H and continue to help confine the flux.

c) property "f "(core loss) imposes an operating restriction requiring keeping the core loss small enough to result in the overall losses in the device satisfying thermal restrictions. This becomes a more and more serious issue as the operating frequency increases.

d) properties "d" and "e" (hysteresis and residual flux) are not useful and might be termed mildly annoying, (except in permanent magnet applications) but are usually held to reasonable size by selecting appropriate materials. While properties "d" and "e" are generally undesirable in power applications (except for permanent magnets), they are at the core of magnetic memory and recording applications.

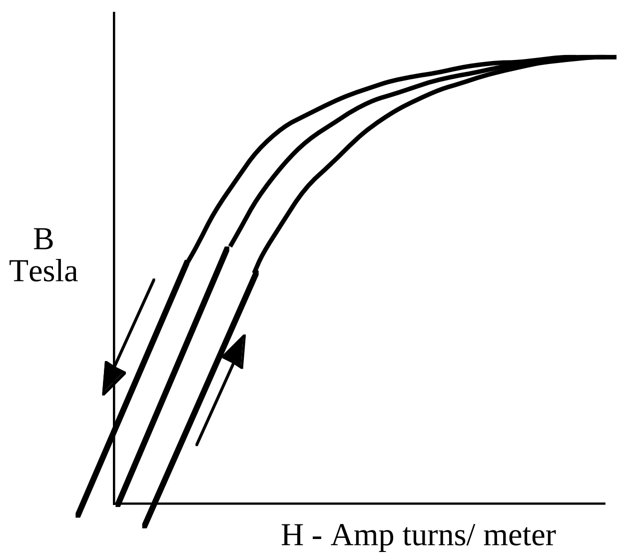
4) <u>Characterization of ferromagnetic material properties</u> - requires representing complex, non linear relationships between B and H and is done in a variety of ways depending on the specific application. For power devices the three important applications are DC devices, AC devices and permanent magnet devices.

a) DC devices - this is the simplest case and requires simply a B vs. H curve as shown below.

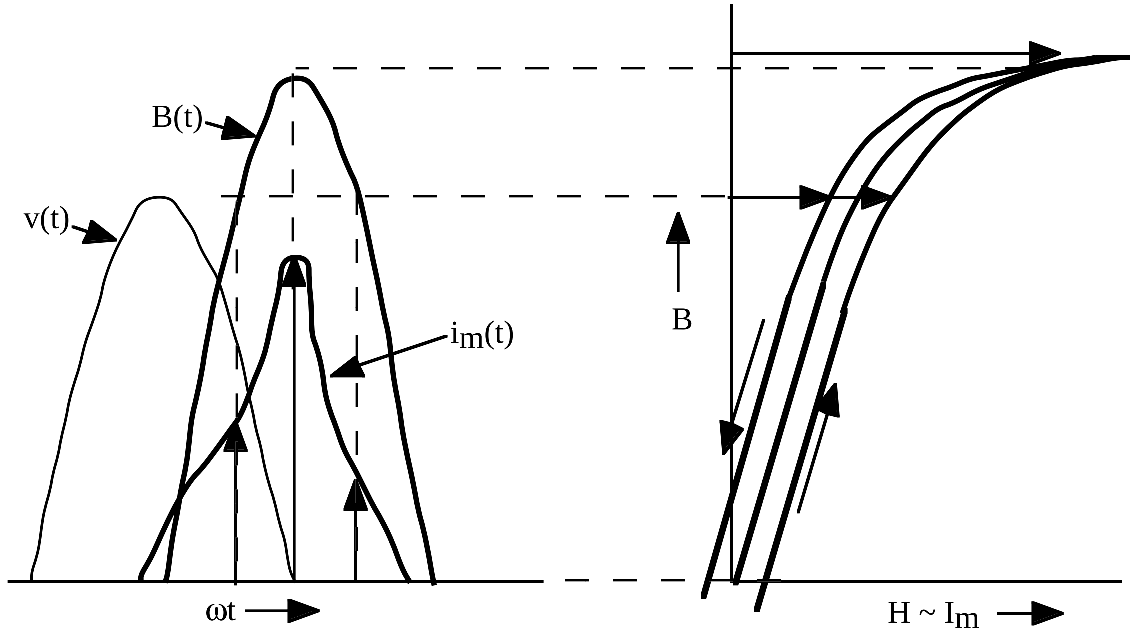


In using such curves it is worth noting that it is generally counterproductive to calculate μ_R since the desired output is B or H and finding their ratio is not useful. μ_R may be useful in comparing two materials but even then the value of B or H is usually acceptable.

b) <u>AC hysteresis loop</u> - the DC B - H curve does not apply to AC because of eddy currents and hysteresis. An attempt to show an AC B-H characteristic similar to the DC case results in a hysteresis loop as shown in the figure below.



This characteristic is not generally useful for power devices since it displays instantaneous, time domain behavior and not magnetizing current and core loss, which are typically the quantities of interest. The complexity of the time domain behavior is shown in the figure below where with sinusoidal voltage and the resulting sinusoidal flux density, the current is non-sinusoidal showing a pronounced peaking. This distortion is created whenever the flux wave pushes the material into the saturated region requiring high magnetizing current. The arrows showing the amplitude of the current wave as found on the hysteresis characteristic at the corresponding flux densities, illustrate how the B(t) and I_m(t) curves are related by the B-H curve.



c) Core loss and exciting volt-amperes - for many power device applications the waveform of the current is itself of very little value. What is required is:

1) the amount of core loss to help evaluate the total losses and ultimately the temperature rise of the device, and

2) the rms value of the current to allow calculation of the I^2R losses in the device and in the circuits feeding the device.

These quantities can be measured on suitable samples of the material (equivalent to calculating the rms current and power associated with the sinusoidal voltage and peaked current in the figure above). The core loss data is typically presented in terms of watts/unit volume or watts/unit weight as a function of flux density B. In like manner the information about rms current is often given in terms of *exciting volt amperes*/unit volume or weight. With a given core at a specified B (plus turns and frequency) Faraday's Law gives the voltage, which can be then used to extract the rms current from the "exciting volt amperes" as read from a curve for the material in question. This current is the ***exciting current*** consisting of a ***core loss component*** associated with the core loss and a **magnetizing component** associated with the creation of the magnetic flux.

These two current components are 90˚ out of phase, the core loss being resistive and the magnetizing current inductive.

The following example will clarify these concepts.

Example - consider an AC magnetic device designed to have the same B everywhere in the core with the following properties

Turns = 350   Core cross section area = 9×10^{-4} (3cm x 3cm)   Core weight = 2.0 lbs

At B = 1.5 T and a frequency = 60 hz, the core loss and exciting VA, read from manufacturers curves, for the core material are

Core loss = 1.85 w/lb   exciting VA = 6.0 VA/lb

Multiplying by the core weight gives the total core loss and exciting VA

Core loss = 3.7 w   exciting VA = 12.0 VA (rms)

Faraday's Law yields the induced voltage as

$$V = 4.44\, f\, N\phi = 4.44 \times 60 \times 350 \times 1.5 \times 9 \times 10^{-4} = 126 \text{ V (rms)}$$

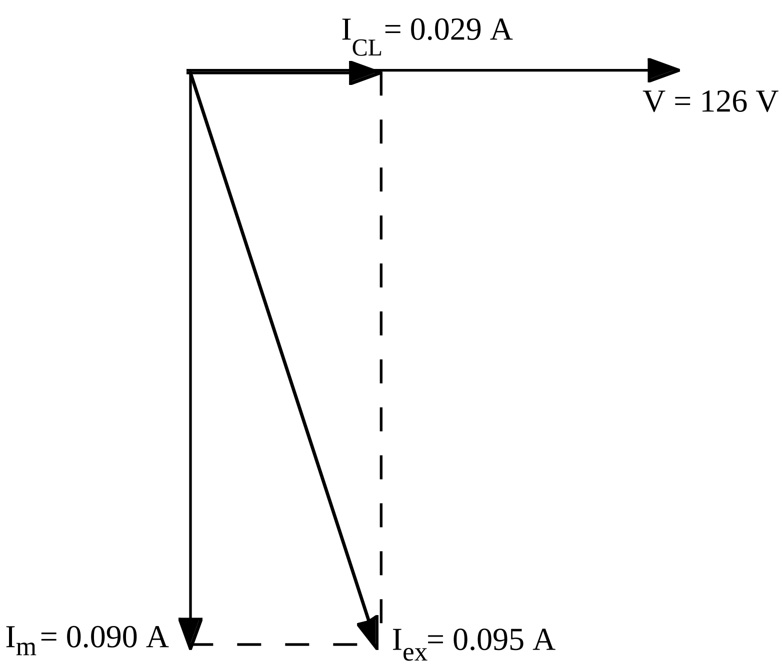
The core loss current and exciting current are then

$$I_{CL} = \frac{3.7}{126} = 0.029 \text{ A (rms)} \qquad I_{ex} = \frac{12.0}{126} = 0.095 \text{ A (rms)}$$

and the magnetizing current is then found from

$$I_m = \sqrt{0.095^2 - 0.029^2} = 0.090 \text{ A (rms)}$$

The current components are shown in phasor form in the diagram below.



5) <u>Inductance - in a linear magnetic system,</u> the flux and magnetizing current are linearly related and the relation is described by the inductance

$$\lambda = N\,\phi = L\,i_m \qquad (10)$$

Faraday's law then becomes

$$\mathbf{v = N\,\frac{d\phi}{dt} = L\,\frac{di_m}{dt}} \qquad (11)$$

From (10) the inductance is the ratio of the flux linkage divided by the magnetizing current required to create the flux

$$\mathbf{L = \frac{\lambda}{i_m} = \frac{N\phi}{i_m}} \qquad (12)$$

In a magnetic system with a well defined flux path and uniform B and H (as is nearly the case in a toroid, for example) and having $B = \mu_0 H$ (only air in the flux path)

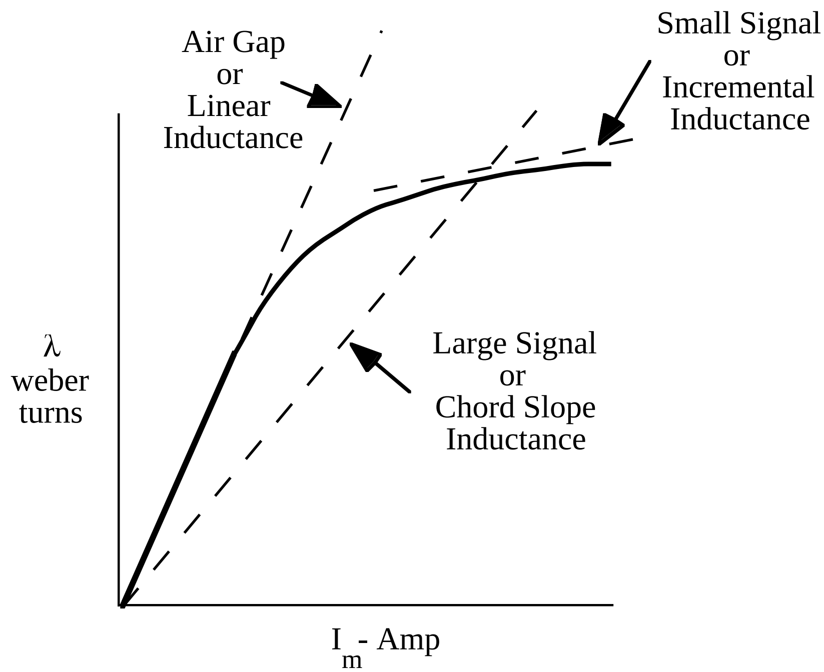
$$L = \frac{N\phi}{i_m} = \frac{N(BA)}{i_m} = \frac{N(\mu_0 HA)}{i_m} = \frac{N\mu_0(Ni_m/\ell)A}{i_m} = \frac{\mu_0 N^2 A}{\ell} \qquad (13)$$

where
L = inductance
N = winding turns
A = cross section area of flux path
ℓ = length of flux path

The inductance is thus a geometric property of a linear magnetic system, depending directly on the effective area through which the flux passes, inversely on the length of the effective magnetic path length and on the square of the coil turns.

$$\mathbf{L = \frac{\mu_0 N^2 A}{\ell}} \qquad \mathbf{(13)}$$

6) <u>Inductance - in systems using ferromagnetic material</u> the flux and magnetizing current are not linearly related and the inductance concept as set forth in the previous section, strictly speaking, does not apply. One major difficulty is illustrated in the figure in section 4b where it is demonstrated that the nonlinearity of the λ vs. i_m curve results in non-sinusoidal current when the flux is sinusoidal. This is clearly not possible for a constant value of L.



As shown in the figure above, attributing a constant inductance to a system using ferromagnetic materials is only possible for low values of λ (low values of B) where the λ vs. i_m characteristic is nearly linear. Most inductors using ferromagnetic materials are designed with air gaps to provide a linear region up and somewhat beyond the desired operating magnetizing current.

There are two other "inductances" often associated with magnetic devices using ferromagnetic materials:

1) <u>the small signal or incremental inductance</u> related to the tangent slope of the λ vs. i_m curve at a specific value of the magnetizing current i_m. As illustrated in the figure above, it represents a small variation $\Delta\lambda$ resulting from a small variation Δi_m about a bias level established by the average value of i_m.

$$\mathbf{L_{inc} = \frac{\Delta\lambda}{\Delta i_m} = \frac{N\Delta\phi}{\Delta i_m}} \qquad \textbf{at bias current } \mathbf{i_{m0}} \qquad \textbf{(14)}$$

It is useful in characterizing filter inductors for DC power supplies and other such devices having large DC currents in the inductor.

2) the "chord slope inductance" related to the slope of a straight line from the origin to the peak value of λ. For DC systems it provide an approximate means for calculating energy storage and time constants. In AC systems it provides a sinusoidal magnetizing current representing the actual nonlinear magnetizing current. For most purposes having the rms values of the two currents the same is desirable, this can be achieved by providing the magnetizing current scale in terms of rms current.

$$L_{chd} = \frac{\lambda_0}{i_{m\,0}} = \frac{N\phi_0}{i_{m\,0}} \qquad \text{at peak flux linkage } \lambda_0 \qquad (15)$$

The "chord slope inductance" is an adequate approximation for many magnetic systems in which the magnetizing current is only a small part of the device total current. Transformers, motors and generators fall in this category in which the magnetic field enables the desired process but does not actively participate in the energy exchange. For these systems the ideal would be zero magnetizing current and good design practice comes close to this ideal. The degree of approximation associated with the "chord slope inductance" is quite acceptable for these systems.

7) DC vs AC Excitation - The difference between DC and AC excitation of a magnetic system can be more clearly explained by utilizing the concept of inductance. In a DC system the relationships between voltage, current and flux are

$$V = I_m R \quad \rightarrow \quad I_m \leftrightarrow NI_m \leftrightarrow \phi \qquad (16)$$

In a well designed AC system the resistance is typically small enough to be neglected (except for the I^2R loss) and the corresponding voltage, current, flux relations are

$$V = I_m (\omega L) \quad \rightarrow \quad I_m \leftrightarrow NI_m \leftrightarrow \phi \qquad (17)$$

which at first sight seems essentially the same as for the DC case. However, the very significant difference between the two becomes apparent if the definition of the inductance (12) is substituted into (17)

$$V = I_m \left(\omega \frac{N\phi}{I_m}\right) = \omega N \phi \quad \rightarrow \quad \phi \leftrightarrow NI_m \leftrightarrow I_m \qquad (18)$$

Canceling the I_m's, the voltage equation reverts to Faraday's Law, yielding a direct means of determining the flux. The physics is the same, the current I_m is still the source of the mmf and flux, but in the AC case it is the flux itself that generates the voltage, not a separate IR drop as for DC.

As a result the applied voltage to an AC magnetic device is realistically a demand for flux and the magnetizing current I_m takes on whatever value needed to create that flux.

The inductance is a convenient means to represent this flux induced voltage for systems without significant magnetic saturation. When there is saturation the magnetizing current is nonsinusoidal (for sinusoidal voltage) and the use of an inductance to model the system, is at best, an approximation. The chord slope inductance is sometimes used to generate an equivalent rms sinusoid for modeling purposes.

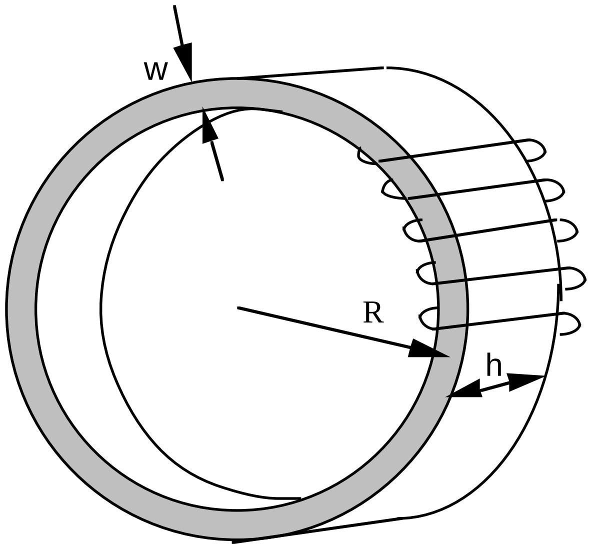
8) The magnetic circuit concept **-** A useful analogy between Ampere's Law for a magnetic circuit and Kirchoff"s law for an electric circuit can be established. Expanding Ampere's Law

$$\text{mmf} = NI_m = H\ell = \frac{B}{\mu}\ell = (BA)\frac{\ell}{\mu A} = \phi\mathcal{R} \tag{19}$$

where

$$\mathcal{R} = \frac{\ell}{\mu A} \quad \ell = \text{path length} \quad A = \text{cross section area for flux} \quad \mu = \mu_0\mu_r = \text{permeability} \tag{20}$$

The toroid in the figure below illustrates the concept.



$\ell = 2\,\pi\,R$ $\qquad$ $A = h\,w$

For comparison, Kirchoff's Law takes the form

$$\text{emf} = V = E\,\ell = \frac{J}{\sigma}\,\ell = (JA)\,\frac{\ell}{\sigma A} = I\,R \qquad (21)$$

where

$R = \dfrac{\ell}{\sigma A}$ $\quad$ ℓ = path length $\quad$ A = cross section area for current $\quad$ σ = conductivity

Comparing the two equations illustrates the analogy

mmf = NI_m ~ V = emf flux = ϕ ~ I = current reluctance = $\mathcal{R}$ ~ R = resistance (22)

In principle the analogy brings to magnetics all of the insight and computational power of electric circuit theory. For example, an air gap in the toroid creates a magnetic circuit with two reluctances, which, by analogy to the electric circuit, are magnetically in series (carry the same flux).

The magnetic circuit analogy and the reluctance concept are invaluable for visualizing and thinking about magnetic systems. When the magnetic system is dominated by air gaps and can be considered linear, the use of reluctance for computation as well as visualization is often an excellent approach. However, when one or more of the reluctances represents ferromagnetic material, the reluctance concept becomes computationally awkward. The problem resides in evaluating μ from the characteristic curves for the material. Since

$$\mu = \frac{B}{H} \qquad (23)$$

one must know either B or H initially to be able to use the curve and find μ. Typically B is known and H is obtained from the appropriate curve. The reluctance can then be found using the core area and the length of the ferromagnetic component. Ultimately the mmf = NI_m will be found by multiplying the total reluctance by the flux.

$$\text{mmf} = NI_m = \mathcal{R}\,\phi = \frac{\ell}{\mu A}(B\,A) \tag{24}$$

Canceling the A's and noting that B/μ is H shows that the mmf is much more directly found from

$$\text{mmf} = NI_m = H\,\ell \tag{25}$$

with no need to mix in the area A or find μ because starting with B we directly find H from the ferromagnetic curve or from μ_0 in the case of air gaps. Using reluctance we find μ and introduce the core area, and then effectively undo both operations to get back to mmf.

Thus for actual calculations the reluctance concept is often the long way around. It is, however, an invaluable aid to thinking about and visualizing magnetic systems.