Problem Set 4

Prob 1

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix} = B \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & -2 & 3 \\ -2 & -5.5 & 5 \\ -3 & -3.5 & 5 \end{bmatrix}$$

The desired basis vectors are therefore $\begin{bmatrix} -2 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -5.5 \\ -3.5 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 5 \end{bmatrix}$

Prob 2

Using Matlab, we find that

$$e^{A+B} = \begin{bmatrix} 3.6439 & 7.9420 & -13.4377 \\ 0 & 0.0498 & 0 \\ 6.7188 & 6.9450 & -9.7938 \end{bmatrix}$$

whereas

$$e^{A} e^{B} = \begin{bmatrix} 0.8780 & 7.3905 & -27.2777 \\ 0 & 0.0498 & 0 \\ 2.3523 & 3.8323 & -10.8970 \end{bmatrix}$$

These are clearly not equal.

In the second case,

$$e^{A+B} = e^{A} e^{B} = \begin{bmatrix} 13.7781 & 12.7781 & -12.7781 \\ 0 & 0.0498 & 0 \\ 6.3891 & 5.4388 & -5.3891 \end{bmatrix}$$

Notice that in the second case, A and B commute, i.e.,

$$AB = BA = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 2 & 0 \\ 1 & 4 & -2 \end{bmatrix}$$

whereas in the first case, A and B do not commute,

$$AB \neq BA$$

As mentioned in class, if A and B commute then $e^{A+B} = e^A e^B$.

Prob 3

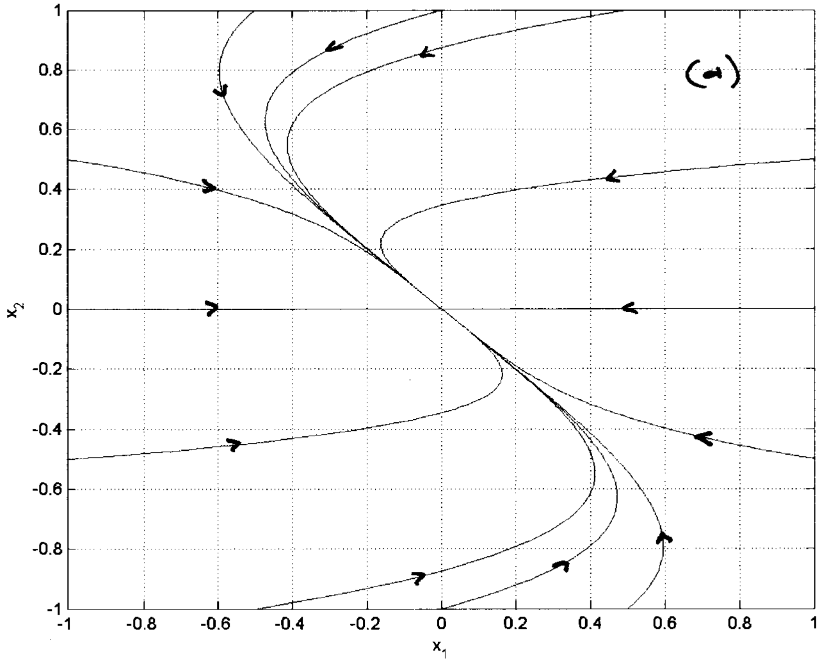
a) $\lambda_1 = -8$, $\lambda_2 = -2$, $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

b) $\lambda_1 = -8$, $\lambda_2 = 2$, $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$

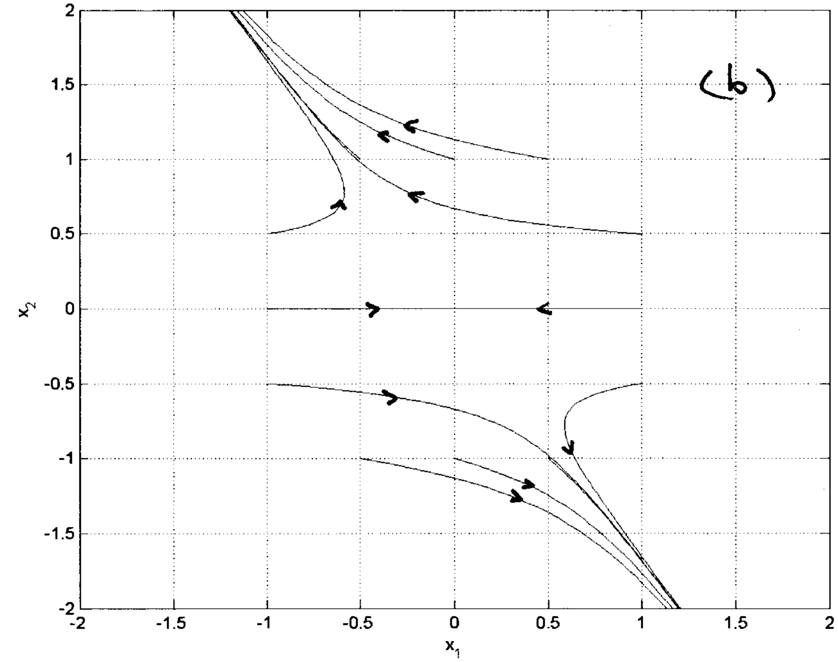
c) $\lambda_1 = -4$, $\lambda_2 = -1$, $v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

d) $\lambda_1 = 0$, $\lambda_2 = -1$, $v_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

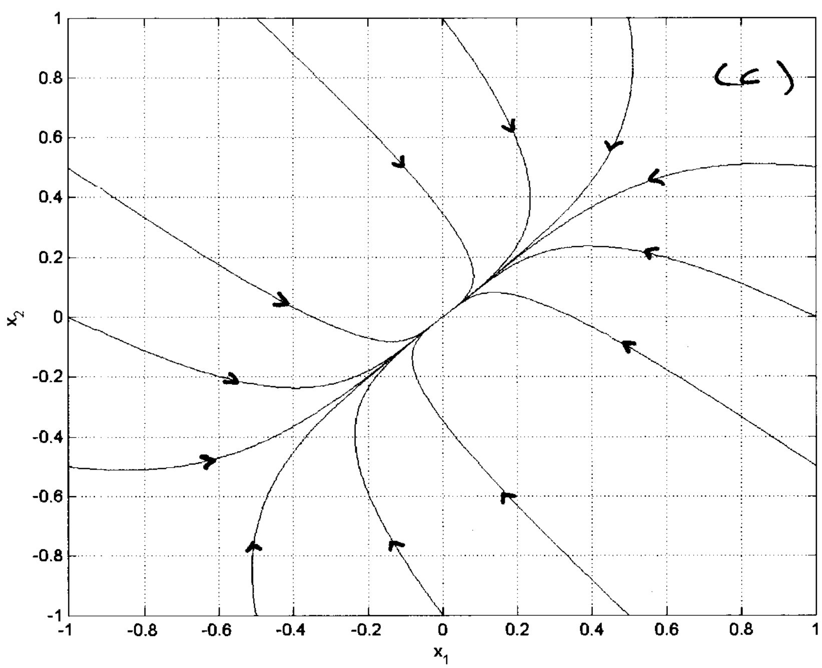
Note that in this case, trajectories stop when they encounter the space spanned by the vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (the vertical axis).



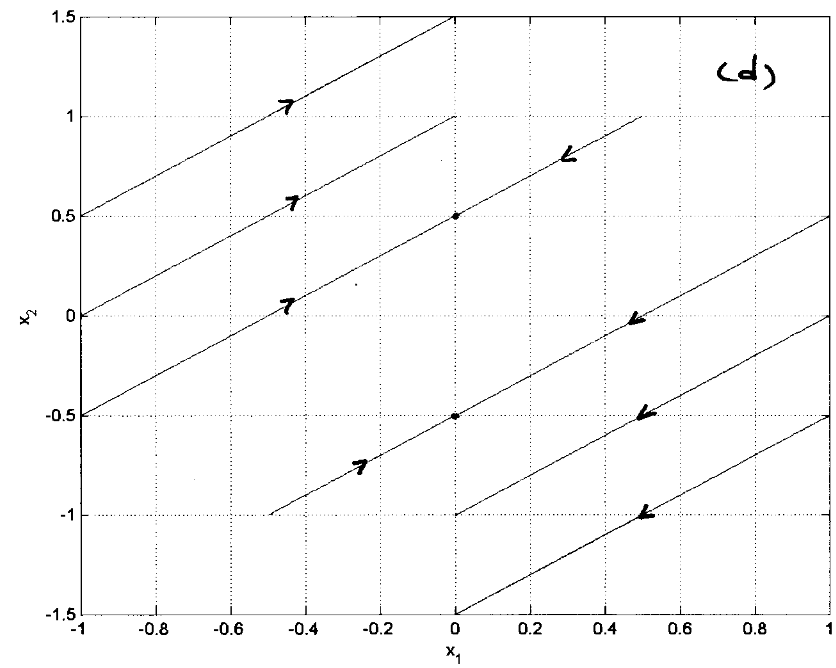
(a)



(b)



(c)



(d)