University of Wisconsin - Madison
Department of Electrical and Computer Engineering

ECE334 - State Space Systems Analysis
Spring 2006

Problem Set 4

Distributed: Monday, 6 March, 2006
Due: Tuesday, 21 March, 2006

Problem 1

In the standard basis, three vectors have representations

$$x_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \quad x_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

The representations of the same vectors in a different basis are

$$\tilde{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \tilde{x}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad \tilde{x}_3 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}.$$

Determine the basis vectors of this different basis.

Problem 2

Let

$$A = \begin{bmatrix} 3 & 2 & -2 \\ 0 & -1 & 0 \\ 1 & -1 & 0 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Show that $e^{A+B} \neq e^A e^B$.

Now let

$$A = \begin{bmatrix} 3 & 2 & -2 \\ 0 & -1 & 0 \\ 1 & -1 & 0 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 0 \\ 1 & 0 & -2 \end{bmatrix}.$$

Show that in this case $e^{A+B} = e^A e^B$.

Why does the equality work in this second case, but not in the first?

Problem 3

Sketch the phase portraits for the systems with the following A-matrices:

$$\text{a)} \begin{bmatrix} -8 & -6 \\ 0 & -2 \end{bmatrix} \qquad \text{b)} \begin{bmatrix} -8 & -6 \\ 0 & 2 \end{bmatrix} \qquad \text{c)} \begin{bmatrix} -2.5 & 1.5 \\ 1.5 & -2.5 \end{bmatrix} \qquad \text{d)} \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}.$$

In each case, determine the eigenvalues/eigenvectors of A, and use that information to guide your sketch.

Confirm your sketches by using Matlab to plot the phase portrait behaviour in each case.