University of Wisconsin - Madison
Department of Electrical and Computer Engineering

ECE334 - State Space Systems Analysis
Spring 2006

Problem Set 3

Distributed: Tuesday, 21 February, 2006
Due: Thursday, 2 March, 2006

Problem 1

Find a basis for the subspace of $\mathbb{R}^4$ that consists of vectors $x = [x_1\ x_2\ x_3\ x_4]^T$ for which $x_1 + x_2 + x_3 = 0$ and $x_3 + x_4 = 0$. (Hint: Write these equations in matrix form, and use null space concepts.)

Problem 2

Bay, problem 3.4

Problem 3

Bay, problem 3.6

Problem 4

Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

Check that $\lambda_1 + \lambda_2 + \lambda_3$ equals the trace, and $\lambda_1\lambda_2\lambda_3$ equals the determinant.

Problem 5

Show that the determinant equals the product of the eigenvalues by imagining that the characteristic polynomial is factored into

$$\det(A - \lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda)\cdots(\lambda_n - \lambda),$$

and making a clever choice of λ.

(See over)

Problem 6

Factor the following matrices into their diagonalized form $S\Lambda S^{-1}$:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$$

Perform this factorization by calculating the eigenvalues and eigenvectors by hand. Show all working. (Matlab provides a useful cross-check though.)

Problem 7

Find the matrix A whose eigenvalues are 1 and 4, and whose eigenvectors are $\left[\begin{smallmatrix} 3 \\ 1 \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} 2 \\ 1 \end{smallmatrix}\right]$ respectively.