Solutions HW #10

Prob 1

The given system is observable but not controllable. It is therefore not a minimal realization. To obtain a minimal realization, find the corresponding transfer function and reduce to its lowest order by cancelling factors common to the numerator and denominator.

Using ss2tf, the corresponding transfer function is

$$H(s) = \frac{3s^2 + 11s + 8}{s^3 + 7s^2 + 14s + 8}$$

$$= \frac{(3s+8)\cancel{(s+1)}}{(s+4)(s+2)\cancel{(s+1)}}$$

$$= \frac{3s+8}{s^2+6s+8}$$

Forming the controllable canonical realization of this transfer function gives.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 8 & 3 \end{bmatrix} x$$

This system is both controllable and observable. It is therefore a minimal realization, as desired.

Prob 2 The controllability (and observability) matrices are both full rank, so the system is controllable (and observable.)

Step 1: transform the system to controllable canonical form.
- The procedure in the notes gives $U = \begin{bmatrix} 17 & 13 & 2 \\ 14 & 9 & 1 \\ 15 & 10 & 1 \end{bmatrix}$

$$\Rightarrow \quad \dot{x}_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{bmatrix} x_c + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad = A_c x_c + B_c u$$

Step 2: the desired characteristic polynomial is

$$(s+3)(s+2+j)(s+2-j) = (s+3)(s^2+4s+5)$$
$$= s^3 + 7s^2 + 17s + 15$$

Therefore the A-matrix of the controlled system is,

$$A_c + B_c K_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -17 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8+k_0 & -14+k_1 & -7+k_2 \end{bmatrix}$$

$$\Rightarrow \quad -8 + k_0 = -15 \quad \Rightarrow \quad k_0 = -7$$
$$-14 + k_1 = -17 \quad \Rightarrow \quad k_1 = -3$$
$$-7 + k_2 = -7 \quad \Rightarrow \quad k_2 = 0$$
$$\Rightarrow \quad K_c = [-7 \quad -3 \quad 0]$$

Step 3: transform K_c back to the original system basis

$$K = K_c U^{-1} = [\tfrac{2}{3} \quad -1\tfrac{2}{3} \quad \tfrac{1}{3}]$$

The closed loop system is then described by,

$$\dot{x} = (A+BK)x = \frac{1}{3}\begin{bmatrix} 4 & -7 & -4 \\ 11 & -8 & -8 \\ 14 & -2 & -17 \end{bmatrix} x$$

The eigenvalues of $A+BK$ are -3 and $-2 \pm j$ as desired.