HW 10
Solutions
p. 1

MP7.6 The MATLAB script to generate the root locus for each controller in parts (a)-(c) is shown in Figure MP7.6. The performance region is indicated on each root locus in Figures MP7.6b - MP7.6d. For part (a), the controller gain is found to be $G_c(s) = 11.3920$. The integral controller in part (b) is determined to be

$$G_c(s) = \frac{4.093}{s} .$$

```
numg=[1]; deng=[1 5 6];
t=[0:0.1:15];
%
% Part (a)
%
num=numg; den=deng;
rlocus(num,den), grid
hold on
plot([-0.4 -0.4],[-10 10],'--',...
[0 -10*tan(36.2*pi/180)],[0 10],'--',...
[0 -10*tan(36.2*pi/180)],[0 -10],'--')
hold off
rlocfind(num,den), pause
%
% Part (b)
%
numc=[1]; denc=[1 0];
[num,den]=series(numc,denc,numg,deng);
rlocus(num,den), grid
hold on
plot([-0.4 -0.4],[-10 10],'--',...
[0 -10*tan(36.2*pi/180)],[0 10],'--',...
[0 -10*tan(36.2*pi/180)],[0 -10],'--')
hold off
rlocfind(num,den), pause
%
% Part (c)
%
numc=[1 1]; denc=[1 0];
[num,den]=series(numc,denc,numg,deng);
rlocus(num,den), grid
hold on
plot([-0.4 -0.4],[-10 10],'--',...
[0 -10*tan(36.2*pi/180)],[0 10],'--',...
[0 -10*tan(36.2*pi/180)],[0 -10],'--')
hold off
rlocfind(num,den)
```

```
»Select a point in the graphics window
selected_point =
  -2.5030 + 3.3380i
ans =
  11.3920
```

```
Select a point in the graphics window
selected_point =
  -0.6690 + 0.8210i
ans =
  4.0930
```

Plot performance region boundaries on graph.

```
Select a point in the graphics window
selected_point =
  -2.0695+ 2.7387i
ans =
  9.2516
```

FIGURE MP7.6
(a) Script to generate the root locus for each controller.

The proportional integral (PI) controller in part (c) is

$$G_c(s) = \frac{9.2516(s+1)}{s} .$$

The **proportional controller** is stable for all $K > 0$ but has a significant st
state error. The **integral controller** has no steady-state error, but is stable

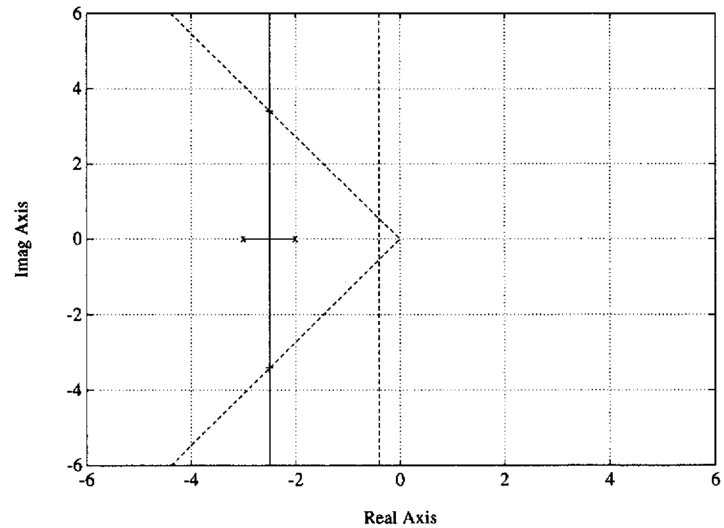

FIGURE MP7.6
CONTINUED: (b) Root locus for proportional controller with selected $K = 11.3920$.

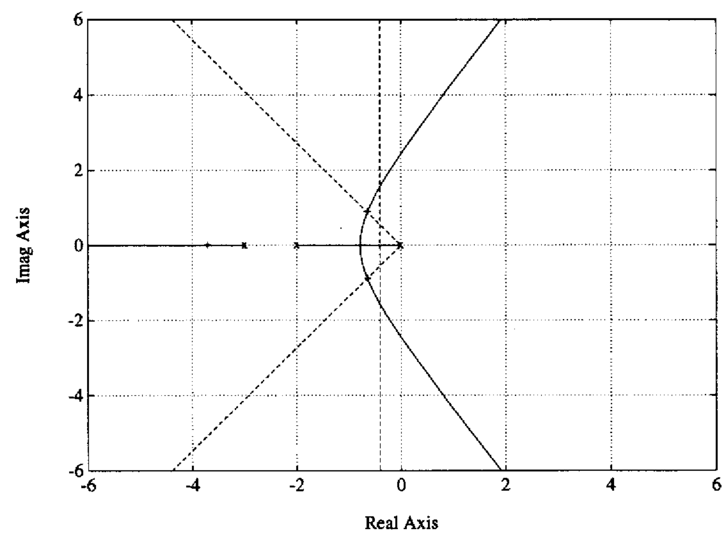

FIGURE MP7.6
CONTINUED: (c) Root locus for integral controller with selected $K = 4.0930$.

for $K < 30$. The **PI controller** has zero steady-state error and is stable for all $K > 0$. Additionally, the PI controller has a fast transient response. The step responses for each controller is shown in Figure MP7.6e.

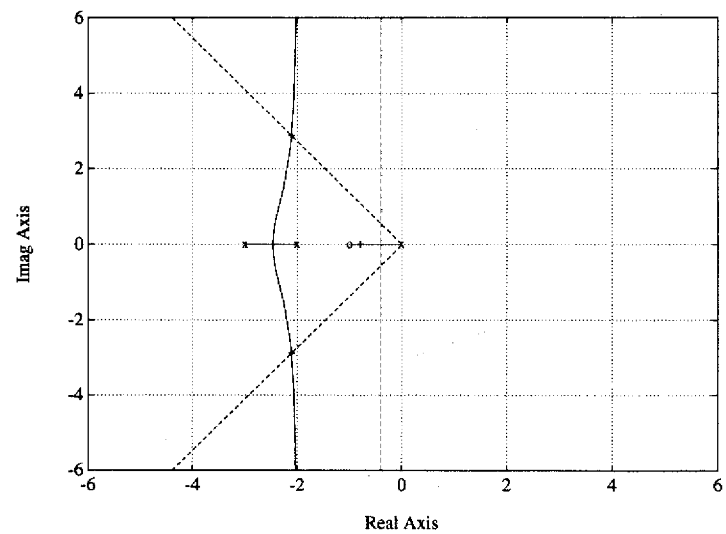

FIGURE MP7.6
CONTINUED: (d) Root locus for PI controller with selected $K = 9.2516$.

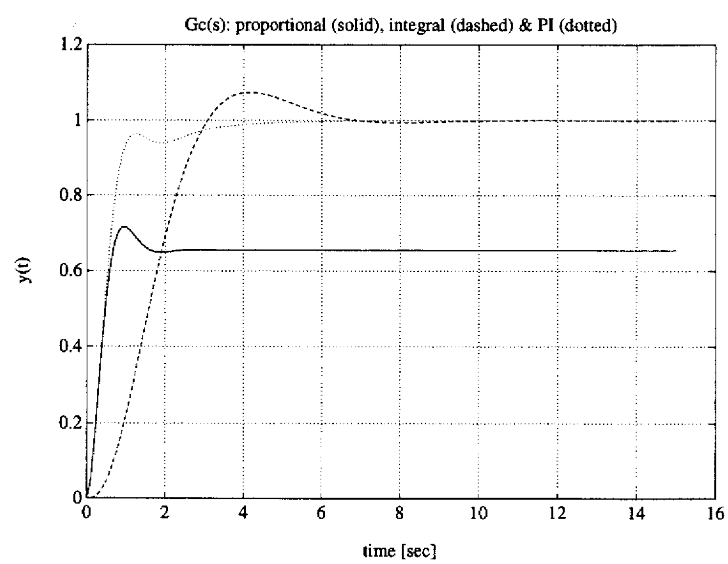

FIGURE MP7.6
CONTINUED: (e) Step responses for each controller.

MP7.7 The open-loop transfer function can be written as

$$G(s) = \frac{K_1 + K_2 s}{J s^2} = \bar{K}_2 \frac{s+5}{s^2}$$

P8.3 (a) The bridged-T network we found has zeros at

$$s = \pm j\omega_n$$

and poles at

$$s = -\frac{\omega_n}{Q} \pm \omega_n\sqrt{1/Q^2 - 1}\,.$$

The frequency response is shown in Figure P8.3 for $Q = 10$.

(b) For the twin-T network, we evaluate the magnitude at

$$\omega = 1.1\omega_n$$

or 10% from the center frequency (see Example 8.4 in Dorf & Bishop). This yields

$$|G| \approx 2.1 \times \left(\frac{0.1}{3.9}\right) \times 1.1 = 0.05\,.$$

Similarly, for the bridged-T network

$$|G| = 2.1 \times \left(\frac{0.1}{2.1}\right) \times 0.14 = .014$$

The bridged-T network possesses a narrower band than the twin-T network.

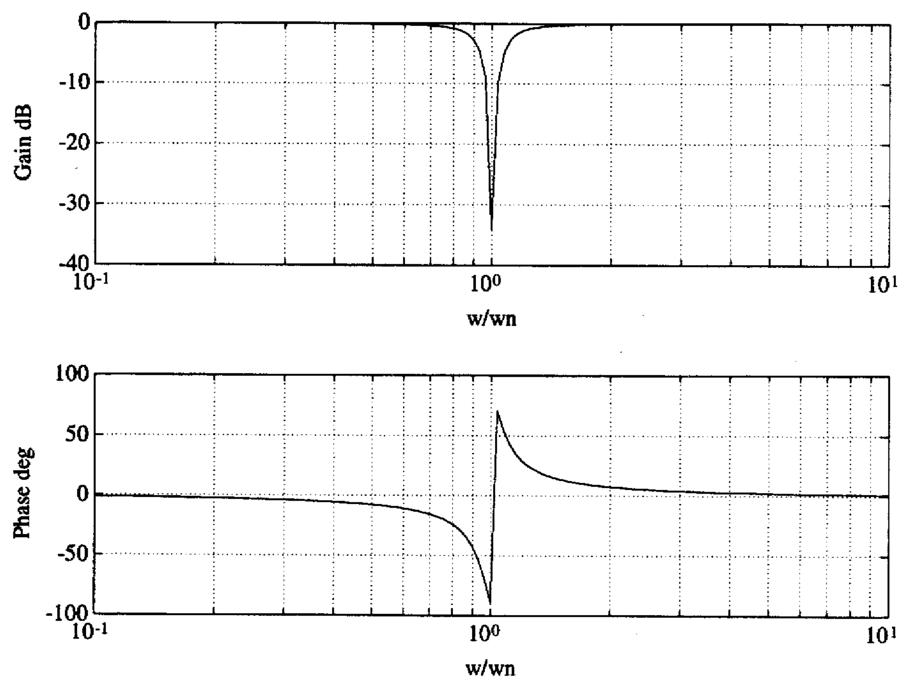

FIGURE P8.3
Bode plot for $G(s) = \frac{s^2+\omega_n^2}{s^2+(2\omega_n/Q)s+\omega_n^2}$, where $\zeta = 1/Q = 0.1$.

P8.8 The transfer function is

$$T(s) = \frac{K}{s^2 + 5s + K} .$$

(a) When $P.O. = 10\%$, we determine that $\zeta = 0.59$ by solving

$$0.10 = e^{-\pi\zeta/\sqrt{1-\zeta^2}} .$$

Then,

$$M_{p\omega} = (2\zeta\sqrt{1-\zeta^2})^{-1} = 1.05 .$$

(b) For second-order systems we have

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} = 0.55\omega_n$$

when $\zeta = 0.59$.

(c) We estimate ω_B (using Figure 8.26in Dorf & Bishop) to be

$$\omega_B \approx 1.1\omega_n .$$