Name: SOLUTIONS

QUIZ #2

1. The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by

$$\frac{dy(t)}{dt} + 500y(t) = 10^4 \int_{-\infty}^{\infty} \frac{d^2x(\tau)}{d\tau^2} z(t-\tau)\, d\tau, \qquad z(t) = te^{-20t}u(t)$$

$$z(t) = te^{-20t}u(t) \;\updownarrow\; Z(\omega) = \frac{1}{(j\omega+20)^2}$$

a) Find the frequency response $H(\omega)$ of this system.

FOURIER TRANSFORMING:

$$j\omega Y(\omega) + 500Y(\omega) = 10^4 (j\omega)^2 X(\omega) Z(\omega)$$

or

$$(j\omega + 500)\, Y(\omega) = \frac{10^4 (j\omega)^2}{(j\omega + 20)^2} X(\omega).$$

Hence,

$$\boxed{H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{10^4 (j\omega)^2}{(j\omega+500)(j\omega+20)^2}}$$

b) For this system, write a linear, constant-coefficient differential equation in terms of $x(t)$ and $y(t)$ not containing integrals.

Simply reverse the process:

$$(j\omega+500)(j\omega+20)^2\, Y(\omega) = 10^4 (j\omega)^2 X(\omega)$$

or

$$(j\omega+500)\left[(j\omega)^2 + 40j\omega + 400\right] Y(\omega) = 10^4 (j\omega)^2 X(\omega)$$

or

$$\left[(j\omega)^3 + 540(j\omega)^2 + 20{,}400 j\omega + 200{,}000\right] Y(\omega) = 10^4 (j\omega)^2 X(\omega)$$

Inverse transforming,

$$\boxed{\frac{d^3y(t)}{dt^3} + 540\frac{d^2y(t)}{dt^2} + 20{,}400\frac{dy(t)}{dt} + 20{,}0{,}000\, y(t) = 10{,}000 \frac{d^2x(t)}{dt^2}}$$

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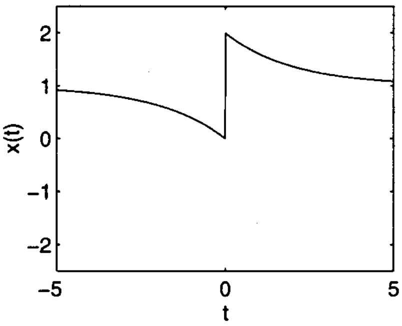
2. Find the Fourier transform of the following signal $x(t)$:



$$x(t) = 1 + e^{-\frac{1}{2}t}u(t) - e^{\frac{1}{2}t}u(-t)$$

$$\Big\updownarrow \mathcal{F}$$

$$\boxed{X(\omega) = 2\pi\delta(\omega) + \frac{1}{\frac{1}{2}+j\omega} - \frac{1}{\frac{1}{2}-j\omega}}$$