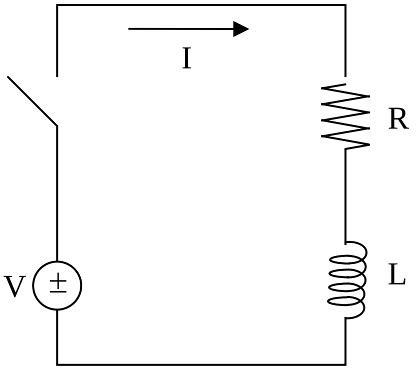
Project 2: Determine current / voltage relationships in RLC circuits by finding I(Ω) and performing an inverse Fourier transform numerically.

PROBLEM 1: RL CIRCUIT



$$V = V_R + V_L$$

$$v[n] = i[n]R + L(i[n] - i[n-1])$$

V = u[n] – u[n-501]

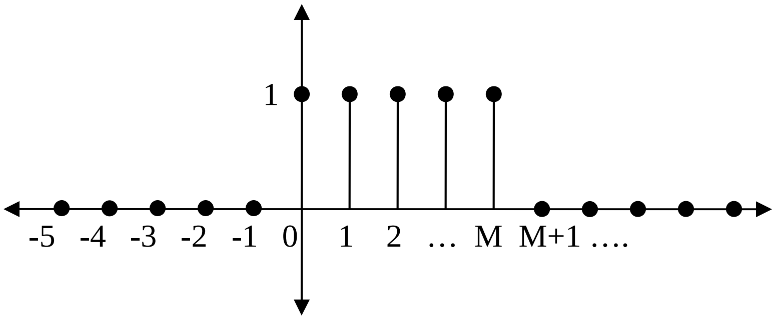
$\Delta t = 10^{-7}$ seconds

[This is the same 50 microsecond square pulse, but discretized]

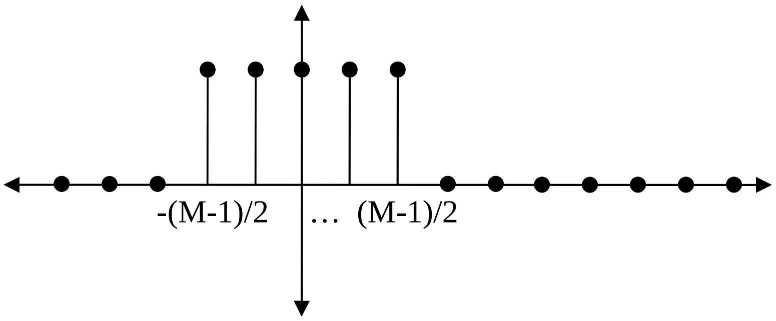
R = 200 Ω $\quad$ $L = 10^{-3}$ Henries

For this project you are given V(Ω). Here is how it was derived.

> Notice we are now in discrete time. We have sampled our 50 microsecond pulse 501 times. This number will change as we change dt, with interesting results.



> V[n] is a time-shifted version of the square pulse found on the FT pairs table. In other words V[n+(M-1)/2] =



Thus

$$V(\Omega) = e^{j\Omega(M-1)/2} \frac{\sin\left(\Omega\frac{M-1}{2}+\frac{1}{2}\right)}{\sin(\Omega/2)} = e^{j\Omega M/2}e^{-j\Omega/2}\frac{\sin(\Omega M/2)}{\sin(\Omega/2)}$$

$$= e^{j\Omega M/2}e^{-j\Omega/2}\frac{e^{j\Omega M/2}-e^{-j\Omega M/2}}{e^{j\Omega/2}-e^{-j\Omega/2}} = \frac{e^{j\Omega M/2}\left(e^{j\Omega M/2}-e^{-j\Omega M/2}\right)}{e^{j\Omega/2}\left(e^{j\Omega/2}-e^{-j\Omega/2}\right)} = \frac{\left(e^{j\Omega M}-1\right)}{\left(e^{j\Omega}-1\right)}$$

You are also given H(Ω) for an RL circuit in the script. Here is how it was derived.

$$v[n] = i[n]R + L(i[n]-i[n-1])$$

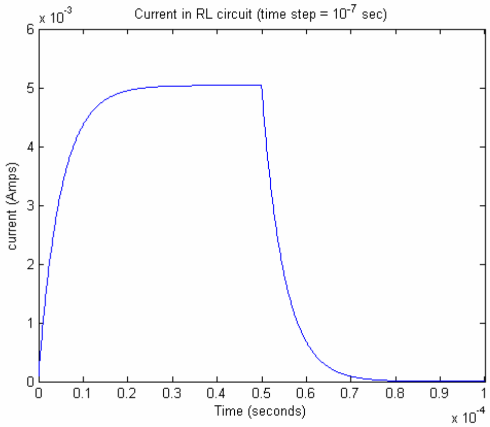
$$H(\Omega) = \frac{I(\Omega)}{V(\Omega)} = \frac{1}{R+\left(1-e^{-j\Omega}\right)L} = \frac{1}{L+R-Le^{-j\Omega}} = \frac{1}{L+R}\frac{1}{1-\frac{L}{L+R}e^{-j\Omega}}$$

The script lets Matlab multiply H(Ω)V(Ω). This is advantageous, since we will be changing both H and V independently.

Your assignment for problem 1 is:

1. Use the InvFT_RLcircuit.m script to transform I(Ω) back into i[n]. The sample code contains all of the information above. All you have to do is run it and print the plot.

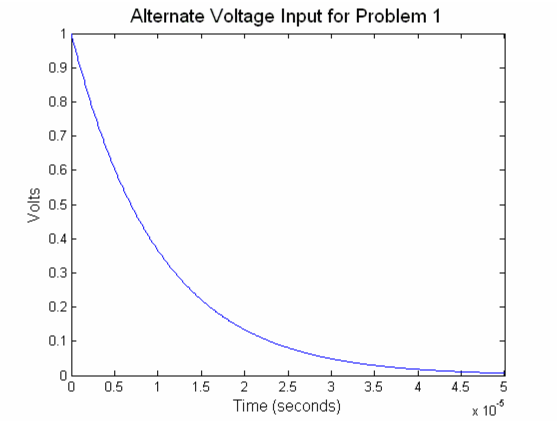
   Your answer should resemble the answer from project 1:



2. The Nyquist theorem, in essence, asserts that when our sampling rate is lowered, more high frequency components of our signal are lost. Show that this is the case by plotting I(Ω) with dt = 10^{-6} (a slower rate) and 10^{-5} (a much slower rate).

3. Instead of a square pulse for the voltage input, use $V[n] = a^n u[n]$, where a=0.995. It looks like this (in continuous time):

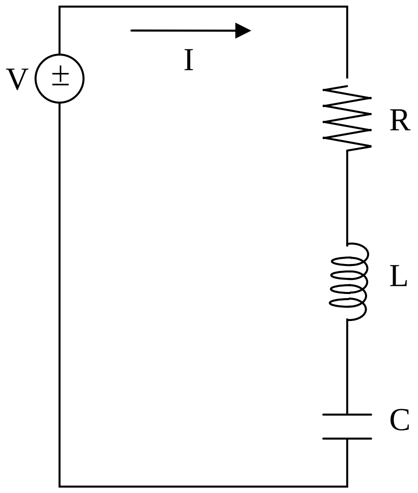
   The transform is on your discrete FT tables. Plug that in for V(Ω) and print the current plot.



FOR PART 1, TURN IN

1. Plot of i(t) for default parameters. This is figure 2.
2. Plot of I(Ω) for dt = 10^{-7} ,10^{-6} and 10^{-5}. This is figure 1. Explain how your plots show the elimination of high-frequency components as the sampling rate decreases.
3. Plot of i(t) with the decaying input.

PROBLEM 2: RLC CIRCUIT



$$V = V_R + V_L + V_C$$

$$v[n] = i[n]R + L(i[n] - i[n-1]) + \frac{1}{C}\sum_{k=-\infty}^{n} i[k]$$

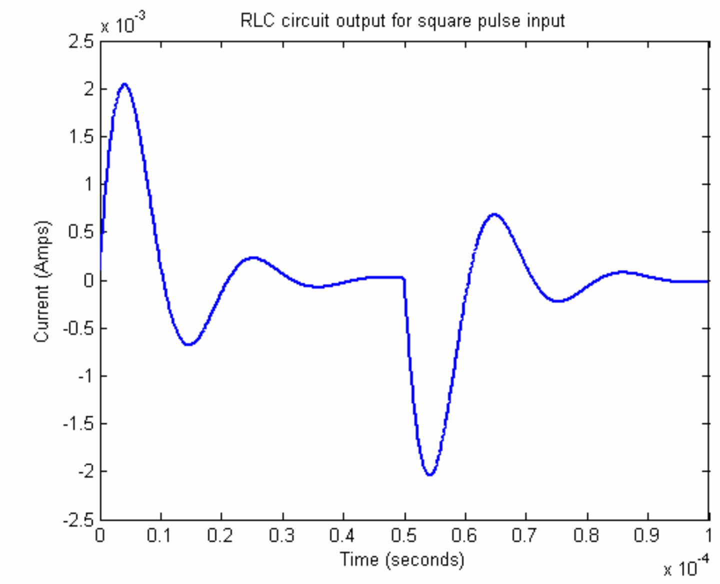
R = 200 Ohms

L = 10^{-3} Henries

C = 10^{-8} Farads

1. Find H(W) and enter it into the script.

2. Print and turn in current plots for the square pulse input and the decaying input. The first of these plots should look like the extra credit problem from the first project:



PROBLEM 3: EXTRA CREDIT

Ben's script uses loops to perform the Inverse Fourier Transform. Mine uses matrix multiplication. Show mathematically that these two routines are doing the same thing.

<u>Ben's Method:</u>

```
Loop over $n_1$, $n_2$ … $n_M$
    Loop over $\Omega_1$, $\Omega_2$ ... $\Omega_Q$
        $y(n_\#) = y(n_\#) + Y(\Omega_\#)e^{i\Omega\#\ n\#}$
    End Ω loop
End n loop
```

<u>Greg's Method:</u>

$$\left[y(n_1) \quad y(n_2) \quad \ldots \quad y(n_M)\right] = \left[Y(\Omega_1) \quad Y(\Omega_2) \quad \ldots \quad Y(\Omega_Q)\right] \times \begin{bmatrix} e^{i\Omega_1 n_1} & e^{i\Omega_1 n_2} & \cdots & e^{i\Omega_1 n_M} \\ e^{i\Omega_2 n_1} & e^{i\Omega_1 n_1} & & \\ \vdots & & \ddots & \\ e^{i\Omega_Q n_1} & & & e^{i\Omega_Q n_M} \end{bmatrix}$$