

FOURIER SERIES

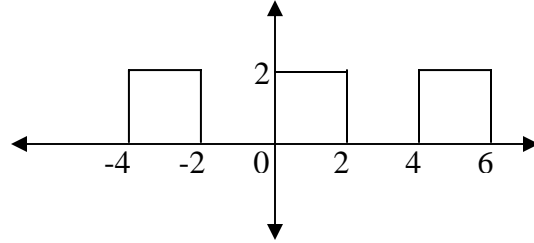
Name: _____

For the periodic signal $f(t)$ shown here, assuming $f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$, solve for

5 pts

- a. The fundamental frequency
- ω_0

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$$

**15 pts**

- b. The constant coefficient
- a_0

$$\frac{1}{T} \int_T \cos(m\omega_0 t) f(t) dt = \frac{1}{T} \int_T \cos(m\omega_0 t) \left[\sum_{n=0}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right] dt$$

$$\frac{1}{4} \int_0^4 \cos(m\omega_0 t) f(t) dt = \frac{1}{4} \int_0^4 a_m \cos^2(m\omega_0 t) dt$$

Set $m=0$

$$\int_0^4 f(t) dt = a_0 \int_0^4 dt \rightarrow a_0 = \frac{1}{4} \int_0^4 2 dt = 1$$

15 pts

- c. All other cosine coefficients
- a_n

$$\frac{1}{4} \int_0^4 \cos(m\omega_0 t) f(t) dt = \frac{1}{4} \int_0^4 a_m \cos^2(m\omega_0 t) dt = \frac{a_m}{2}$$

$$a_m = \frac{1}{2} \int_0^2 2 \cos(m\omega_0 t) dt = \frac{1}{m\omega_0} \sin(m\omega_0 t) \Big|_0^2 = \frac{\sin(m\pi)}{m\omega_0} = 0$$

15 pts

- d. All sine coefficients
- b_n

$$\frac{1}{T} \int_T \sin(m\omega_0 t) f(t) dt = \frac{1}{T} \int_T \sin(m\omega_0 t) \left[\sum_{n=0}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right] dt$$

$$\frac{1}{4} \int_0^4 \sin(m\omega_0 t) f(t) dt = \frac{1}{4} \int_0^4 b_m \sin^2(m\omega_0 t) dt = \frac{b_m}{2}$$

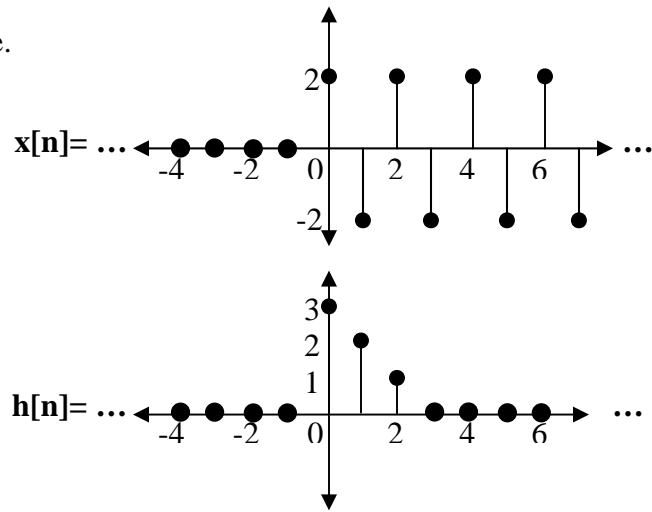
$$b_m = \frac{1}{2} \int_0^2 2 \sin(m\omega_0 t) dt = -\frac{1}{m\omega_0} \cos(m\omega_0 t) \Big|_0^2 = \frac{1}{m\omega_0} [1 - \cos(m\pi)]$$

$$b_{\text{odd}} = \frac{4}{m\pi}, \quad b_{\text{even}} = 0$$

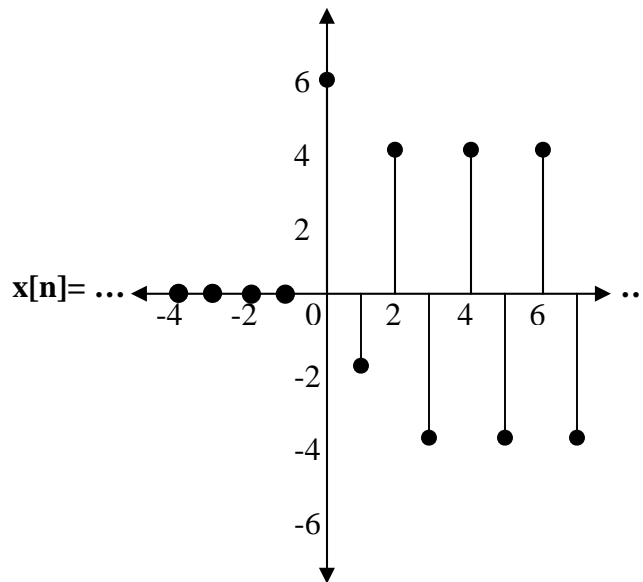
CONVOLUTION

Graph $x[n]*h[n]$ for the discrete-time signal and impulse response shown here.

25 pts

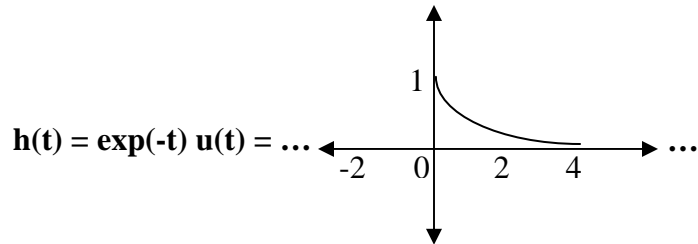
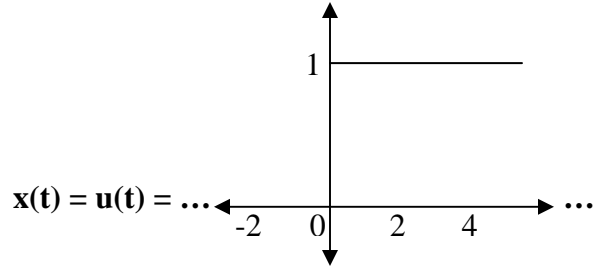


One way to make this easier is to remember that $x[n]*h[n] = h[n]*x[n]$, so your answer becomes $y[n] = 3x[n] + 2x[n-1] + x[n-2]$.



Graph $x(t)*h(t)$ for the continuous-time signal and impulse response shown here. Make sure to show your work for the convolution integral calculation.

25 pts



Again you can make this problem simpler by switching $x(t)$ and $h(t)$, so that the convolution integral becomes

$$y(t) = \int_{-\infty}^{\infty} \exp(-\tau)u(\tau)u(t-\tau)d\tau = \int_0^t \exp(-\tau)d\tau = [-\exp(-\tau)]_0^t = 1 - \exp(-t)$$

