Discussion 03/06/07.

Continuous time

Periodic

Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

Period T

$$\omega_0 = 2\pi/T$$

$$x(t) \xleftrightarrow{F.S.} a_k$$

$(k\omega_0)$

Non-periodic.

Fourier Transform.

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

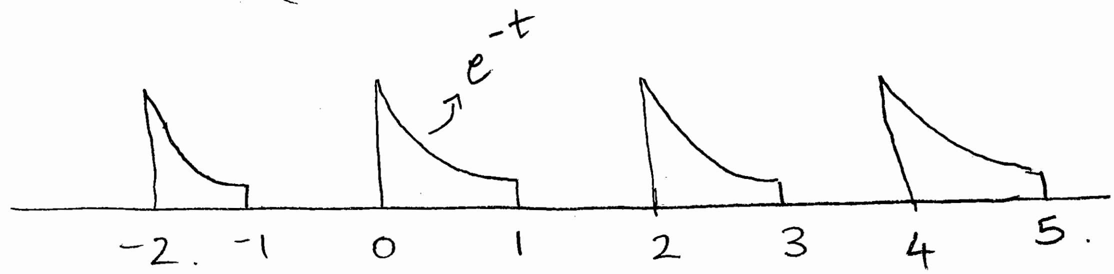
$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) \xleftrightarrow{F.T.} X(j\omega)$$

Fourier Transform of a periodic signal.

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0).$$

3.50 (e). Find Fourier Series coefficients of $x(t)$.



$T = 2$. $\quad \omega_0 = \frac{2\pi}{2} = \pi$

$$a_0 = \frac{1}{2}\int_0^2 x(t)\,dt = \frac{1}{2}\int_0^1 e^{-t}\,dt.$$

$$= \frac{1}{2}\left[1 - e^{-1}\right] \approx 0.316.$$

$$a_k = \frac{1}{2}\int_0^2 x(t)\,e^{-jk\pi t}\,dt = \frac{1}{2}\int_0^1 e^{-t}.e^{-jk\pi t}\,dt$$

$$= \frac{1}{2}\int_0^1 e^{-(1+jk\pi)t}\,dt = \frac{1}{2}\left[\frac{e^{-(1+jk\pi)t}}{-(1+jk\pi)}\right]_0^1$$

$$= \frac{1}{2(1+jk\pi)}\left[1 - e^{-(1+jk\pi)}\right].$$

$$= \frac{1}{2(1+jk\pi)}\left[1 - e^{-1}(-1)^k\right]$$

↳ since $e^{-j\pi} = -1$

What is the Fourier Transform of $x(t)$ given in 3.50 (e) ?

$$X(j\omega) = \sum_{k=-\infty}^{\infty} a_k \, \delta(\omega - k\omega_0) . 2\pi$$

$$= \sum_{k=-\infty}^{\infty} \frac{\pi}{(1+jk\pi)} \left[1 - (-1)^k e^{-1}\right] . \delta(\omega - k\pi)$$

3.54 (b). Find the Fourier Transform of $x(t) = e^{-4|t|}$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-4|t|} e^{-j\omega t} \, dt.$$

$$= \int_{-\infty}^{0} e^{4t} e^{-j\omega t} \, dt + \int_{0}^{\infty} e^{-4t} e^{-j\omega t} \, dt.$$

$$= \int_{-\infty}^{0} e^{(4-j\omega)t} \, dt + \int_{0}^{\infty} e^{-(4+j\omega)t} \, dt.$$

$$= \left[\frac{1 - 0}{(4-j\omega)}\right] + \left[\frac{-0+1}{(4+j\omega)}\right]$$

$$X(j\omega) = \frac{1}{4-j\omega} + \frac{1}{4+j\omega}.$$

$$= \frac{4+j\omega + 4-j\omega}{4^2 - (j\omega)^2} = \frac{8}{16+\omega^2}$$

Check Fourier Transform pairs.

Page 774. Table C.4

$$x(t) = e^{-a|t|} \quad a > 0.$$

$$\updownarrow \text{F.T.}$$

$$X(j\omega) = \frac{2a}{a^2+\omega^2}.$$

Fourier transform of $x(t) = \delta(t)$

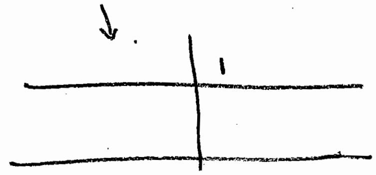
$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt.$$

Remember $$\int_{-\infty}^{\infty} \delta(t-k)\, f(t)\, dt = f(k).$$

So $$\int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt. = \int_{-\infty}^{\infty} \delta(t-0)\, e^{-j\omega t}\, dt.$$

$$= e^{-j\omega(0)} = 1.$$

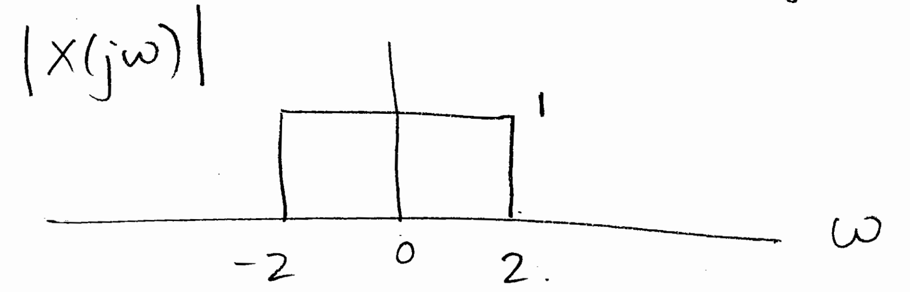
$$\delta(t) \xleftrightarrow{F.T.} 1$$

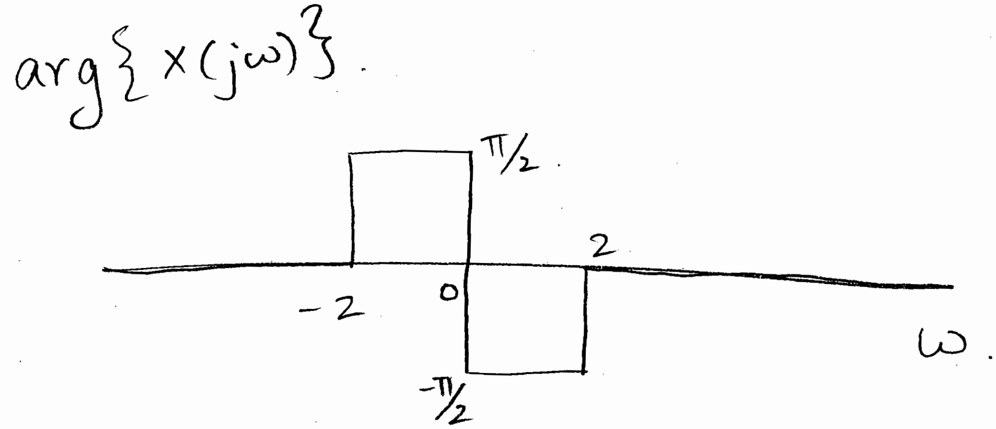


$x(t)$	$X(j\omega)$.
infinite $\longleftrightarrow$	finite.
finite $\longleftrightarrow$	infinite.

[Except for periodic $x(t)$

3.55 (f). Find $x(t)$ from $X(j\omega)$.





$$X(j\omega) = |X(j\omega)|\, e^{j(\arg\{X(j\omega)\})}.$$

So.

$$X(j\omega) = 1\, e^{-j\pi/2} \qquad 0 \le \omega \le 2.$$

$$= 1\, e^{j\pi/2} \qquad -2 \le \omega < 0.$$

$$= 0 \qquad \text{elsewhere.}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi} \left[\int_{0}^{2} e^{-j\pi/2} e^{j\omega t}\, d\omega + \int_{-2}^{0} e^{j\pi/2} e^{j\omega t}\, d\omega \right]$$

$$= \frac{1}{2\pi} \left[e^{-j\pi/2} \int_{0}^{2} e^{j\omega t}\, d\omega + e^{j\pi/2} \int_{-2}^{0} e^{j\omega t}\, d\omega \right]$$

$$= \frac{1}{2\pi} \left\{ e^{-j\pi/2} \left[\frac{e^{j\omega t}}{jt} \right]_{0}^{2} + e^{j\pi/2} \left[\frac{e^{j\omega t}}{jt} \right]_{-2}^{0} \right].$$

$$e^{j\pi/2} = j$$
$$e^{-j\pi/2} = -j$$

$$= \frac{1}{2\pi} \left\{ \frac{e^{-j\pi/2}}{jt} \left[e^{j2t} - 1 \right] + \frac{e^{j\pi/2}}{jt} \left[1 - e^{-j2t} \right] \right]$$

$$= \frac{1}{2\pi t} \left[1 - e^{j2t} + 1 - e^{-j2t} \right].$$

$$= \frac{1}{2\pi t} \left[2 - \left(e^{j2t} + e^{-j2t} \right) \right].$$

$$= \frac{1}{2\pi t} \left\{ 2 - 2\cos 2t \right]$$

$$x(t) = \frac{1 - \cos 2t}{\pi t}$$