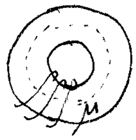
Problem #1



According to Example 6-14

$$\vec{B} = B_\phi \hat{a}_\phi \quad dl = \hat{a}_\phi \, r d\phi$$

$$\oint_c B \cdot dl = \int_0^{2\pi} B_\phi \, r d\phi = 2\pi B_\phi r$$

$$2\pi r B_\phi = \mu_0 N I$$

$$B_\phi = \frac{\mu_0 N I}{2\pi r}$$

$$\Phi = \int_S B \cdot ds = \int_S \left(\hat{a}_\phi \frac{\mu_0 N I}{2\pi r}\right) \cdot \left(\hat{a}_\phi \, dz \, dr\right)$$

$$= \int_{r=a}^{r=b} \frac{\mu_0 N I h}{2\pi} \frac{1}{r} dr$$

$$= \frac{\mu_0 N I h}{2\pi} \ln\left(\frac{b}{a}\right)$$

flux linkage $\Lambda = N\Phi = \frac{\mu_0 N^2 I h}{2\pi} \ln\left(\frac{b}{a}\right)$

$$\boxed{L = \frac{\Lambda}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)}$$

$$L = \frac{4\pi \times 10^{-7} N^2 (4.8 \times 10^{-3})}{2\pi} \ln\left(\frac{6.6}{3.8}\right) = 10^{-3}$$

$$\boxed{N \approx 1413 \text{ turns}}$$

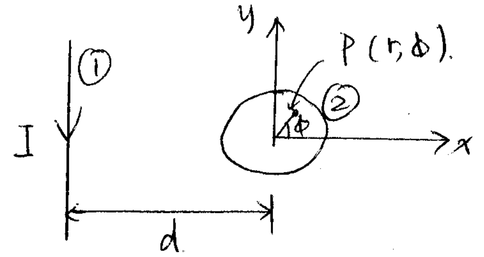
b) $B_\phi = \frac{\mu_0 \mu_r N I}{2\pi r}$

$$L = \frac{\mu_0 \mu_r N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

for $L = 10^{-3}$

$$\boxed{N = 100 \text{ turns}}$$

Problem #2



$\vec{B}_1$ due to straight-line current carrier at point P

$$B_1 = \frac{\mu_0 I}{2\pi r'} \quad \text{(out of paper direction)}$$

where $r' = d + r\cos\phi$

$$B = \frac{\mu_0 I}{d + r\cos\phi} \quad \text{(out of paper direction)}$$

$$\Lambda_{12} = N_2 \Phi_{12}$$

$$= N_2 \int_{S_2} B_1 \cdot ds \qquad \text{where } N_2 = 1$$

$$= \int_{S_2} B_1 \cdot ds$$

$$= \frac{\mu_0 I}{2\pi} \int_{S_2} \frac{1}{d + r\cos\phi}\, r\,dr\,d\phi \qquad \hat{a}_z : \text{out of paper}$$

$$= \frac{\mu_0 I}{2\pi} \int_{r=0}^{r=b} \left(\int_{\phi=0}^{\phi=2\pi} \frac{d\phi}{d + r\cos\phi} \right) r\,dr$$

According to Integration Table

$$\int \frac{d\phi}{d + r\cos\phi} \quad \text{for } d^2 > r^2$$

$$= \frac{2}{\sqrt{d^2 - r^2}} \tan^{-1}\left[\sqrt{\frac{d-r}{d+r}} \tan\frac{\phi}{2} \right] + C$$

$$\Lambda_{12} = \frac{\mu_0 I}{\cancel{2\pi}} \int_0^b \frac{\cancel{2\pi}\, r\,dr}{\sqrt{d^2 - r^2}}$$

$$= \mu_0 I \left(d - \sqrt{d^2 - b^2} \right)$$

$$L_{12} = \mu_0 \left(d - \sqrt{d^2 - b^2} \right)$$

P.6-27 a) $\mathcal{R}_g = \frac{l_g}{\mu_0 S} = \frac{3\times10^{-3}}{4\pi\times10^{-7}\times(\pi\times0.025)^2} = 1.21\times10^6\ (H^{-1})$,

$$\mathcal{R}_c = \frac{2\pi\times0.08 - 0.003}{3000\times(4\pi\times10^{-7})\times(\pi\times0.025)^2} = 6.75\times10^4\ (H^{-1}).$$

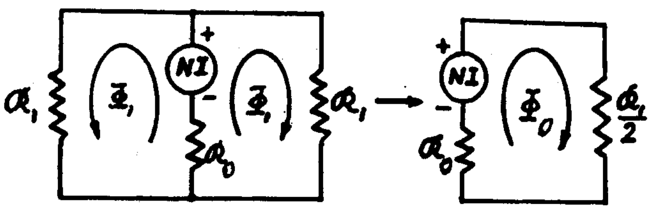
b) $\bar{B}_g = \bar{B}_c = \bar{a}_\phi \frac{10^{-5}}{\pi\times0.025^2} = \bar{a}_\phi\, 5.09\times10^{-3}\ (T)$,

$\bar{H}_g = \frac{1}{\mu_0}\bar{B}_g = \bar{a}_\phi \frac{5.09\times10^{-3}}{4\pi\times10^{-7}} = \bar{a}_\phi\, 4.05\times10^3\ (A/m)$,

$\bar{H}_c = \frac{1}{\mu_0\mu_r}\bar{B}_c = \bar{a}_\phi \frac{4.05\times10^3}{3000} = \bar{a}_\phi\, 1.35\ (A/m)$.

c) $NI = \Phi(\mathcal{R}_c + \mathcal{R}_g)$, $I = \frac{1}{N}\Phi(\mathcal{R}_c + \mathcal{R}_g) = 0.0256\ (A) = 25.6\ (mA)$.

P. 6-28 Magnetic circuit:



$\Phi_1 = \Phi_0/2$

$\frac{1}{\mu_0 S} = \frac{1}{(4\pi10^{-7})\times10^{-3}} = 7.95\times10^8$

Neglecting leakage flux and assuming constant flux density over S:

$\mathcal{R}_0 = \frac{0.002}{\mu_0 S} + \frac{0.24-0.002}{\mu_0\mu_r S} \cong 1.60\times10^6\ (H^{-1})$,

$\mathcal{R}_1 = \frac{0.24+2\times0.2}{\mu_0\mu_r S} = 0.102\times10^6\ (H^{-1})$.

a) $\Phi_0 = \frac{NI}{\mathcal{R}_0 + \mathcal{R}_1/2} = 3.63\times10^{-4}\ (Wb)$; $\Phi_1 = \frac{\Phi_0}{2} = 1.82\times10^{-4}\ (Wb)$.

b) $H_1 = \frac{\Phi_1}{\mu_0\mu_r S} = 28.9\ (A/m)$,

$(H_0)_g = \frac{1}{\mu_0 S}\Phi_0 = 28.9\times10^4\ (A/m)$ in air gap,

$(H_0)_c = (H_0)_g/\mu_r = 57.8\ (A/m)$.

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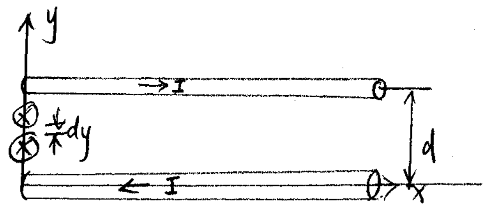

$\vec{B}$ at y due to two current carriers

$$\vec{B} = -\hat{a}_z \frac{\mu_0 I}{4\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right)$$

Remark: use result from Text Book P237

$$\vec{B} = \hat{a}_z \frac{\mu_0 I L}{4\pi r\sqrt{L^2+r^2}} \quad \text{for semi-infinite long} \quad L >> r$$

$$= \hat{a}_z \frac{\mu_0 I}{4\pi r} \quad \text{for } L >> r$$

$$d\vec{l} = \hat{a}_y \, dy$$

$$d\vec{F} = I \, d\vec{l} \times \vec{B}$$

$$= -\hat{a}_x \frac{\mu_0 I^2}{4\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right) dy$$

The way to define the integration value for dy:

I define as followings: the rod is placed / touched just to the end of two current carrides.

Thus: y is integrated from b to d−b

$$\vec{F} = -\hat{a}_x \frac{\mu_0 I^2}{4\pi} \int_b^{d-b} \left(\frac{1}{y} + \frac{1}{d-y}\right) dy$$

$$= -\hat{a}_x \frac{\mu_0 I^2}{4\pi} \left[\ln(y) - \ln(d-y)\right]_b^{d-b}$$

$$= -\hat{a}_x \frac{\mu_0 I^2}{4\pi} \left[2 \ln\left(\frac{d-b}{b}\right)\right]$$

$$= -\hat{a}_x \frac{\mu_0 I^2}{2\pi} \ln\left(\frac{d}{b} - 1\right)$$