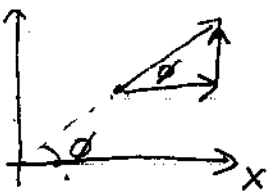
Sample final answers Spring 2002 (1/2)

1a) Because its direction changes along the path of integration $\hat{a}_r$



b) $\hat{a}_r = \hat{a}_x \cos\phi + \hat{a}_y \sin\phi$

c) $d\vec{l}' = r\,d\phi'\,\hat{a}_\phi = a\,d\phi'\,\hat{a}_\phi$

d) $d\vec{l}' \times (-a\hat{a}_r + z\hat{a}_z)$

$= a\,d\phi'(-a\hat{a}_\phi \times \hat{a}_r + z\hat{a}_\phi \times \hat{a}_z)$

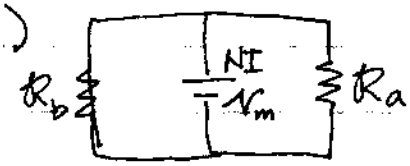
$= a\,d\phi'(a\hat{a}_z + z\hat{a}_r)$

$= a\,d\phi'(a\hat{a}_z + z\cos\phi\,\hat{a}_x + z\sin\phi\,\hat{a}_y)$

$$\vec{B} = \frac{\mu_0 I}{4\pi(a^2+z^2)^{3/2}}\left[a\hat{a}_z\int_0^{2\pi} d\phi + z\hat{a}_x\int_0^{\pi}\cos\phi\,d\phi + z\hat{a}_y\int_0^{\pi}\sin\phi\,d\phi\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi(a^2+z^2)^{3/2}}\left[a\pi\hat{a}_z + z\hat{a}_y\right]$$

2a)



$\mathcal{V}_m = NI$

$\mathcal{R}_a = \frac{a}{\mu_a cW}$

$\mathcal{R}_b = \frac{b}{\mu_b dW}$

b) $\Phi = \frac{\mathcal{V}_m}{\mathcal{R}_b} = \frac{NI\mu_b dW}{b}$

same for i. and ii.

c) downward

d) $L = \frac{N\Phi}{I} = \frac{N^2\mu_b dW}{b}$

2/2

3 a) $|E| = \max$ at $r = 1\,cm$

b) c) $\oint \vec{D} \cdot d\vec{s} = Q \Rightarrow E = \frac{Q}{4\pi\epsilon R^2}$

$E(R=1) = E_{max} = 200\,MV/m = \frac{Q}{4\pi\epsilon(0.01)^2}$

$\Rightarrow \frac{Q}{4\pi\epsilon} = 20\,kV \cdot m$

$$\boxed{E_R = \frac{(20\,kV \cdot m)}{R^2}} \qquad \vec{E} = E_R \hat{a}_R = \frac{20}{R^2} \hat{a}_R \quad (kV/m)$$

b) $V_{max} = -\int \vec{E} \cdot d\vec{\ell} = -\int_{.02}^{.01} \frac{20}{R^2} dR \quad (kV)$

$= 20\left(\frac{1}{0.01} - \frac{1}{0.02}\right)$

$$\boxed{V_{max} = 1\,MV}$$

4 a) $I = \int \vec{J} \cdot d\vec{S} = |J|\,A = 10^3 \pi (2 \times 10^{-3})^2$

$= 2\pi \times 10^{-3}\,A$

b) $\vec{J} = 10^3 \sin\varphi\, \hat{a}_R$

$I = \int \vec{J} \cdot d\vec{S} = \int_0^{2\pi}\int_0^{\pi} 10^3 \sin\phi\, R^2 \sin\theta\, d\theta\, d\phi$

$\Rightarrow I = 0$

5) a) $V_m = NI = 10$ A-turns

b) $\epsilon = \epsilon_r \epsilon_0 = 80\epsilon_0$

c) V = volume enclosed by surface S

d) $\vec{R}$ = vector from ds' to observation point

e) $\vec{J} \neq \sigma\vec{E}$ in vacuum tube, because electrons move collisionlessly