

**ECE 220 Midterm 1**  
**Sample Exam Problems**

**Note: The problems given here represent typical ECE 220 exam problems. However, as a whole, this set of problems is more extensive in length than a typical exam.**

**Problem 1.**

Provide short answers to the following:

- a) Describe the relationship between the surface S and the contour C in Stokes' Theorem.
- b) Determine which of the following are conservative (irrotational) vector fields:

$$\vec{A} = \frac{x}{\sqrt{x^2 + y^2 + z^2}} \hat{a}_x$$

$$\vec{B} = \nabla f \text{ where } f \text{ is an arbitrary scalar field}$$

$$\vec{C} = r \hat{a}_r \text{ (spherical coordinates)}$$

$$\vec{D} = 2\mathbf{r} \cos \mathbf{f} \hat{a}_r - \mathbf{r} \sin \mathbf{f} \hat{a}_\phi \text{ (cylindrical coordinates)}$$

**Problem 2.**

Consider the closed region bounded by the planes  $x = -1$ ,  $x = 1$ ,  $z = 0$ ,  $z = 2$ ,  $y = 2$ , and the cylindrical surface defined by  $y = \sqrt{1 - x^2}$ . This is essentially a cube with a semi-cylindrical "bite" taken out of one side. Find the net flux of  $\vec{A}$  out of this region if

$$\vec{A} = 6rz \hat{a}_r + 2 \frac{z^2}{r} \hat{a}_\phi - 2z \hat{a}_z$$

**Problem 3.**

An infinitely long cylindrical beam of electrons has a radius  $a$  and an electron volume density of

$$n = \begin{cases} 10^{16} e^{-r^2/a^2} & , r \leq a \\ 0 & , r > a \end{cases}$$

What is the average charge density (charge per unit volume) in the beam?

**Problem 4.**

For the scalar field

$$V = -r e^{-z} \sin f ,$$

determine the magnitude and direction of the maximum rate of increase of  $V$  at the point  $P(r, f, z)$ .

**Problem 5.**

Consider an electric field given by  $\vec{E} = -\nabla V$ , where  $V$  is the scalar field from problem 4. If the integral

$$\int_A^B \vec{F} \cdot d\vec{l}$$

represents the work done in moving a particle from point A to point B, find the work done by the electric field on a point charge  $q$  that travels along a straight line from  $A(1, \frac{p}{2}, 0)$  to  $B(1, \frac{p}{2}, 1)$ .

(Points A and B are given in cylindrical coordinates.)