Sample problem set 2.

In all problems. $\varepsilon_0 = 8.854 \times 10^{-12}$ (F/m)

(1) The following vectors are given in free space (Example 4-1 in the textbook)

(a) $A_1 = \hat{x}\, x$

(b) $A_2 = \hat{x}\, 5$

(c) $A_3 = \hat{z}\, 6y$

Are these vector fields conservative? (or electrostatic fields).

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

{curl(A)}

you might need this ↗.

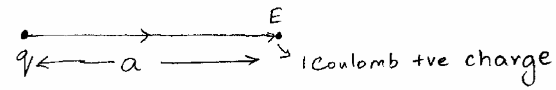
(2) A dielectric slab has an electric field $\vec{E} = 4\hat{x} + 5y\hat{y} + 10x\hat{z}$ (V/m). Find the volume charge (ρ_v) → density in the dielectric if the permittivity of the dielectric $\varepsilon = 4\varepsilon_0$.

(Hint: $\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$) ↗ this might be useful

(3). According to Coulomb's law, the force on a unit positive charge due to a point charge q at a distance 'a' meters is

$$E = \frac{q}{4\pi\varepsilon_0 a^2}$$

($\varepsilon = \varepsilon_0$ in free space or air.)



Can you show this is true, using Gauss's law?

(Hint: start by drawing a Gaussian surface (sphere) of radius 'a' around the point charge q).

(4). In a slab of dielectric material for which $\varepsilon = 2.5\,\varepsilon_0$ & $V = 200\,z^2$ (volts) find

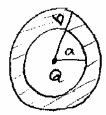
(a) The flux density (D)

(b) volume charge density (ρ_v)

(c) Polarization vector ($\vec{P}$)

(Hint: Use the divergence formula from problem (2) & grad formula from the quiz)

(5) A point charge Q is surrounded by a hollow conducting sphere: Calculate



inner radius – 'a'
outer radius – 'b'

a) Surface charge densities on the inner & outer surfaces. of the sphere

b) Electric fields in the three regions.

(i) $r < a$.

(ii) $a < r < b$

(iii) $r > b$.

(This is a tougher one; draw a Gaussian surface (sphere) in each of the three regions & then use Gauss's law for part b)

(6). An electron of charge 'e' & mass 'm' is placed at a point A, at a distance 'r' meters from a charge Q (+ve charge).

(a) Calculate the acceleration on the electron (direction also)

(b) If we placed the electron at a point B (distance $r_1 > r$), would the acceleration be higher ?

For problem 6, you can use the formula for electric field due to a point charge from problem (3) & Force = mass × acceleration

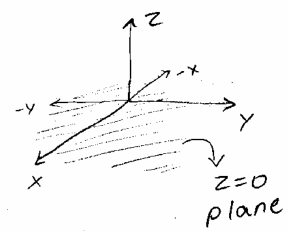
$$\vec{F} = m\vec{a}$$

(7) The plane $z=0$ marks the boundary between free space ($\varepsilon_r = 1$) & a dielectric medium with $\varepsilon_r = 40$. ($\varepsilon = \varepsilon_r \varepsilon_0$)

The E field in the free space, just next to the interface is.

$$\vec{E} = 13\hat{x} + 40\hat{y} + 50\hat{z} \text{ (V/m)}$$

Determine $\vec{E}$ on the other side of the interface (in the dielectric)



Identify the tangential & normal components & then use the boundary conditions

$D = \varepsilon E$ $\quad D_{n_2} - D_{n_1} = \rho_s$

$E_{t_2} - E_{t_1} = 0$.

(assume $\rho_s = 0$)