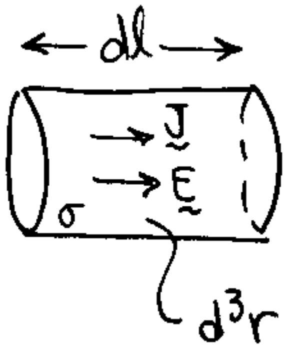
Power Dissipation ≡ Joule's Law

As conduction charges move through a medium, they experience many collisions & friction, agitating the medium's atoms (i.e, "heating" the medium). Consider a volume, d^3r of conducting medium in which a (steady) current is sustained by an electric field:



Q: What is the <u>rate</u> of energy transferred (Joules/s = Watts, power dissipation) by, say, N charges comprising current $\underset{\sim}{J}$ in d^3r?

$$dW = \begin{bmatrix} \text{energy lost by} \\ \text{N charges due} \\ \text{to collisions in} \\ dl \end{bmatrix} = \begin{bmatrix} \text{energy gain by} \\ \text{N charges from} \\ \underset{\sim}{E} \text{ over distance} \\ \underset{\sim}{dl} \end{bmatrix}$$

(assumes steady state! energy balance)

$$dW = N \times (\underset{\sim}{F} \cdot d\underset{\sim}{l}) = (N) q \underset{\sim}{E} \cdot d\underset{\sim}{l}$$

$$= (n\, d^3 r)\, q \underset{\sim}{E} \cdot d\underset{\sim}{l}$$

This event occurs in the time it takes for the N charges to cross distance $d\underset{\sim}{l}$:

$$dt = \frac{d\underset{\sim}{l}}{\underset{\sim}{u}}$$

avg drift velocity of charges accelerated by $\underset{\sim}{E}$ & resisted by $\underset{\sim}{F}_{friction}$

Incremental Energy loss (transfer) rate is therefore:

$$dP = \frac{dW}{dt} = \frac{n\, d^3 r\, q \underset{\sim}{E} \cdot d\underset{\sim}{l}}{dt} = n\, d^3 r\, q \underset{\sim}{E} \cdot \underset{\sim}{u} \quad \left(\underset{\sim}{u} = \frac{d\underset{\sim}{l}}{dt}\right)$$

(power)

or

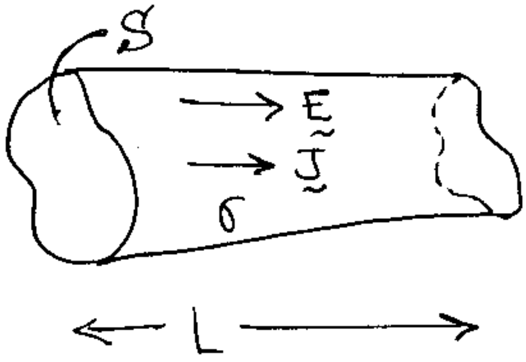
$$dP = \left(q n \underset{\sim}{u} \cdot \underset{\sim}{E}\right) d^3 r = \underset{\sim}{J} \cdot \underset{\sim}{E}\; d^3 r$$

For a total volume, τ, of conducting media, the power dissipation by a conduction current is :

$$\boxed{P(\text{watts}) = \iiint_{\tau} \left(\underset{\sim}{J} \cdot \underset{\sim}{E}\right) d^3 r}$$

Joule's "Law"

NOTE: FOR



if $\underset{\sim}{E}$ is uniform across the cross-sectional area, then

$$P = \iiint \underset{\sim}{J} \cdot \underset{\sim}{E}\, d^3r = \int_L dl\, E \iint_S J\, ds = \Delta V \cdot I \quad !$$

Also, since $\Delta V = IR$,

$$\boxed{P = I^2 R}$$