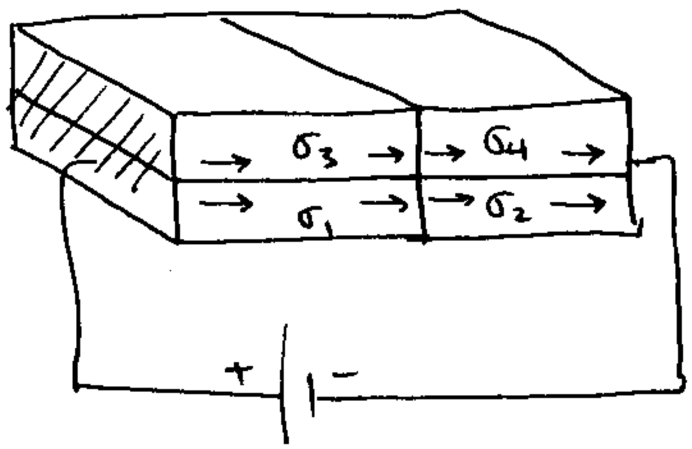
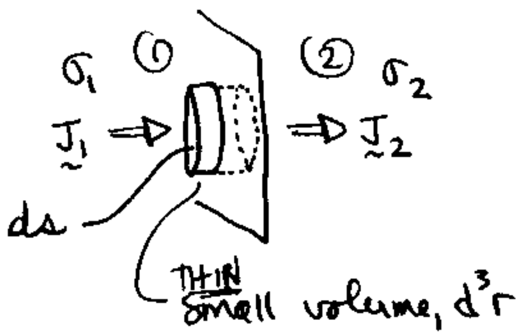
Interface Conditions for $\underset{\sim}{J}$ at boundaries between different conducting media



At interface between ① & ②:



From charge conservation:

$$\nabla \cdot \underset{\sim}{J} = -\frac{\partial \rho_v}{\partial t}$$

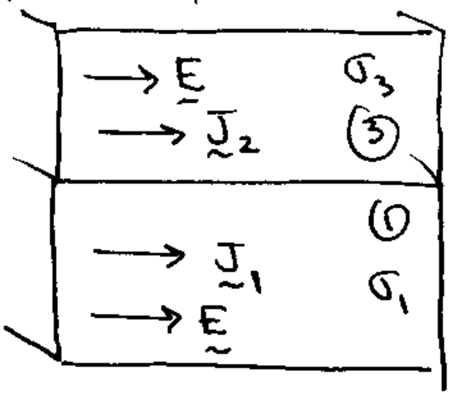
IF steady state, then $\nabla \cdot \underset{\sim}{J} = 0$,

and $$\iiint_{d^3r} \nabla \cdot \underset{\sim}{J} = \oiint_{S_{d^3r}} \underset{\sim}{J} \cdot d\underset{\sim}{s} \approx (J_{2n} - J_{1n}) \times ds = 0$$

$\therefore$ $\boxed{J_{1n} = J_{2n}}$ if st. st.

(IF not steady state, then ρ_s = fcn(time) at interface)

At interface between (1) & (3),



Recall $E_{1t} = E_{2t}$ (to keep $\nabla \times \underset{\sim}{E} = 0$)

$$\Rightarrow \boxed{\frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2}} \quad (\text{for } \underset{\sim}{J} = \sigma \underset{\sim}{E})$$

You may (are encouraged) to read Sections 4.7 & 4.9 in the text on your own.