Energy Stored in a Magnetic Field

If you'll recall previous discussion, we claimed that it is useful and convenient to suggest that energy is stored in an electric field:

$$W_{elec} = \iiint \frac{\underset{\sim}{D} \cdot \underset{\sim}{E}}{2} d^3r = \iiint \frac{\epsilon |\underset{\sim}{E}|^2}{2} d^3r \quad \text{(Joules)}$$

and the energy density at some point, $\underset{\sim}{r}$, is

$$w_e = \frac{\underset{\sim}{D} \cdot \underset{\sim}{E}}{2} = \frac{1}{2} \epsilon |\underset{\sim}{E}(\underset{\sim}{r})|^2 \; (J/m^3).$$

It turns out more difficult to derive an expression showing that energy is stored in a static magnetic field. However, if we were to complete such a derivation, we would obtain:

$$w_m = \frac{\underset{\sim}{B} \cdot \underset{\sim}{H}}{2} = \frac{1}{2} \mu |\underset{\sim}{H}(\underset{\sim}{r})|^2 \quad (J/m^3), \text{ energy density}$$

and

$$W_m = \iiint \frac{1}{2}(\underset{\sim}{B} \cdot \underset{\sim}{H}) d^3r = \iiint \frac{\underset{\sim}{B} \cdot \underset{\sim}{H}}{2} d^3r \quad \text{(Joules)}$$