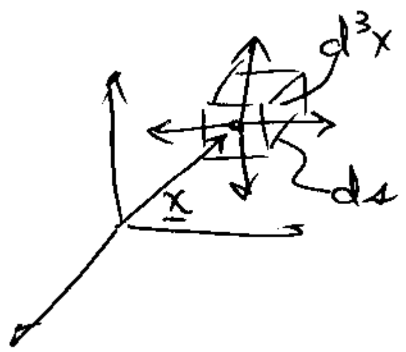
By the way, recall that

$$\nabla\cdot\underset{\sim}{B} \equiv \lim_{V\to d^3x} \frac{\oiint_{S_V} \underset{\sim}{B}\cdot d\underset{\sim}{s}}{V} = \frac{\oiint_{s_{d^3x}} \underset{\sim}{B}\cdot d\underset{\sim}{s}}{d^3x}$$

MULTIPLY BOTH SIDES BY d^3x:

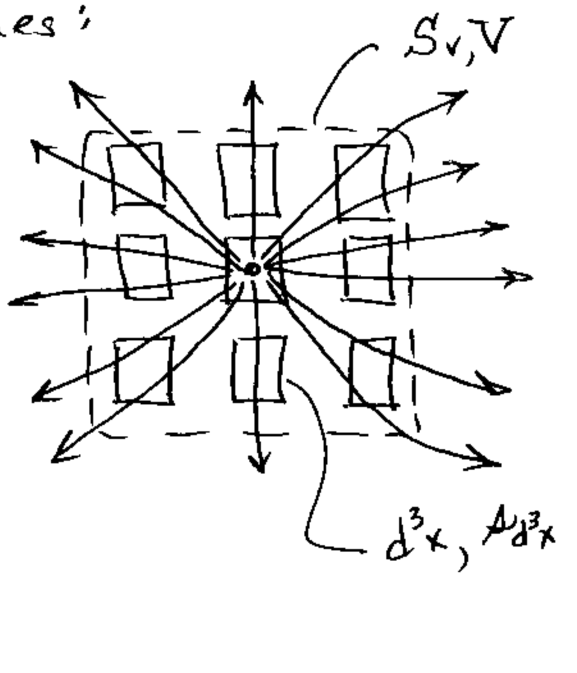
$$\Rightarrow \boxed{\left(\nabla\cdot\underset{\sim}{B}\right)_{\text{at } \underset{\sim}{x}}\cdot d^3x = \oiint_{s_{d^3x}} \underset{\sim}{B}\cdot\hat{n}\, d\underset{\sim}{s}}$$



Consider Stacking Many d^3x's side-by-side.

→ Observe that summing all the net fluxes out of all the neighboring infinitesimal volumes yields the same value as computing the net outward flux through the <u>macroscopic</u> surface that encloses the entire collection of little d^3x volumes:

$$\sum_{\substack{\text{all } s_j \,\& \\ d^3x_j}} \oiint_{s_j} \underset{\sim}{B}\cdot\hat{n}\, ds = \oiint_{S_V} \underset{\sim}{B}\cdot d\underset{\sim}{s}$$



However, using more conventional notation, we recognize that cumulatively adding up the values of $(\nabla\cdot\underset{\sim}{B})_{\text{at } \underset{\sim}{x}_j} d^3x_j$ at all points $\underset{\sim}{x}_j$ within V is written as:

$$\sum_{\substack{\text{all } d^3x_j \\ \text{in } V}} (\nabla\cdot\underset{\sim}{B})_{\underset{\sim}{x}_j} d^3x_j = \boxed{\iiint_V \nabla\cdot\underset{\sim}{B}(\underset{\sim}{x})\, d^3x}$$

Hence it appears that equating the two types of sums:

$$\boxed{\iiint_V (\nabla\cdot\underset{\sim}{B})\, d^3x = \oint\!\!\oint_{S_V} \underset{\sim}{B}\cdot d\underset{\sim}{s}}$$

Divergence Theorem

A tool to convert from one expression to the other, depending on which is most convenient to evaluate for a particular application.