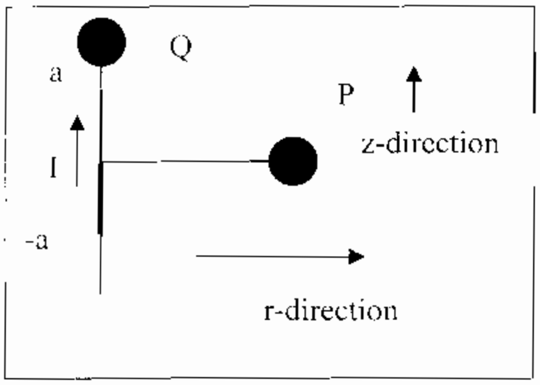
ECE 220 Quiz#3
April 27, 2007

graded by

Name Solutions

Andrew	**Problem 1**	**(20pts)**	________
David	**Problem 2**	**(20pts)**	________
Grader	**Problem 3**	**(25pts)**	________
David	**Problem 4**	**(25pts)**	________
Andrew	**Problem 5**	**(10pts)**	________
	Bonus Problem	**(5pts)**	________
	Total Points		________

1) (20 points) We can model complex magnets by approximating their continuous, arbitrary path (such as HSX coils) as a contiguous set of filamentary sticks of a finite length. Consider the following diagram of one such 'stick' of current in a cylindrical system. Point P is a distance ρ out from the center of the segment and Q is a distance ρ (greater than 'a') out from the origin on the axis of the segment. Use the Biot-Savart law to find the magnetic field at point P.



Specifically, identify the following elements at point P:

$\underline{dl}$ = $dz'\,\hat{z}$

$\underline{r}'$ = $z'\,\hat{z}$

$\underline{r}$ = $\rho\,\hat{r}$

$(\underline{r} - \underline{r}')$ = $\rho\hat{r} - z'\hat{z}$

$|\underline{r} - \underline{r}'|^3$ = $(\rho^2 + z'^2)^{3/2}$

Put these pieces together to set up the integral you need to evaluate   do not simplify here

$\underline{B}$ = $\dfrac{\mu_0}{4\pi}\displaystyle\int_{-a}^{a} \frac{I\,dz'\hat{z} \times (\rho\hat{r} - z'\hat{z})}{(\rho^2 + z'^2)^{3/2}}$ T

Simplify the integrand and pull out constants, but DO NOT EVALUATE

$\underline{B}$ = $\dfrac{\mu_0 I}{4\pi}\displaystyle\int_{-a}^{a} \frac{dz'(\hat{z}\times\rho\hat{r} + \hat{z}\times(-z'\hat{z}))}{(\rho^2+z'^2)^{3/2}} = \frac{\rho\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dz'}{(\rho^2+z'^2)^{3/2}}\,\hat{\phi}$ T

What is the value of $\underline{B}$ at point Q? Why?

0   $d\underline{l}$ is ∥ to $(\underline{r} - \underline{r}')$

2) (20 pts) An infinitely long circular wire of radius "a" (thick wire) has a current density that varies with radius from the center of the wire as $J = J_0 (1 - r/a)$ A/m^2 and zero outside the wire.

Use Ampere's Law to find the magnetic field inside and outside of the wire.

$$\oint \underline{B} \cdot d\underline{l} = \mu_0 I_{enclosed}$$

By symmetry B_ϕ is constant on circle of radius r & in $\hat{\phi}$ dire.

$$2\pi r B_\phi = \mu_0 I_{encl.}$$

$$\underline{J} = J_0 (1 - r/a) \hat{z}$$

$$I_{enc} = \int \underline{J} \cdot d\underline{S} = \int_0^{2\pi} \int_0^r J_0 (1 - r/a)\, r\, dr\, d\phi$$

$$= 2\pi J_0 \int_0^r \left[r - \frac{r^2}{a} \right] dr = 2\pi J_0 \left[\frac{r^2}{2} - \frac{r^3}{3a} \right]_0^r$$

$$= 2\pi J_0 \left(\frac{r^2}{2} - \frac{r^3}{3a} \right) \quad \text{For } r < a$$

$$I_{enc} = 2\pi J_0 \left(\frac{a^2}{2} - \frac{a^3}{3a} \right)$$

$$= \frac{2\pi J_0}{6} a^2 = \frac{\pi J_0}{3} a^2 \quad \text{For } r \geq a$$

$$\underline{B} = \hat{\phi} \frac{\mu_0 I_{enc}}{2\pi r}$$

$0<r<a$ $\underline{B} = \hat{\phi} \frac{\mu_0}{2\pi r} 2\pi J_0 \left(\frac{r^2}{2} - \frac{r^3}{3a} \right) = \mu_0 J_0 \left(\frac{r}{2} - \frac{r^2}{3a} \right) \hat{\phi}$ T

$r > a$ $\underline{B} = \hat{\phi} \frac{\mu_0}{2\pi r} \frac{\pi J_0 a^2}{3} = \frac{\mu_0 J_0}{6r} a^2 \hat{\phi}$ T

What is the total current flowing in the wire?

$$I = \frac{\pi J_0 a^2}{3} \text{ A}$$

3) (25 points) A parallel plate capacitor is formed with two metal plates 50mm by 50mm separated by a glass slab of the same area as the plates and thickness of 'd'. The relative permittivity of the glass is $\epsilon_r = 5$. The upper plate is fixed to the glass and the lower plate is not. The mass of the lower plate is 0.01 kg.

a) First find the energy in the capacitor.

$$W_E = \frac{1}{2} C V^2 \qquad C = \frac{\epsilon A}{d}$$

$$W_E = \frac{\epsilon A V^2}{2d} \ \text{J}$$

b) Realizing that force is the negative gradient of the energy, find the minimum voltage that should be applied between the capacitor plates in order for the lower plate to remain in position against the force of gravity if the thickness of the glass 'd' is 6mm. ($g = 9.8 \ m/s^2$)

BALANCE GRAVITY WITH electROSTATIC FORCE (virt. work)

$$\underline{F} = -\frac{\partial W_E}{\partial d} = + \frac{\epsilon A V^2}{2 d^2} = mg$$

$$V^2 = \frac{mg \, 2 d^2}{\epsilon_r \epsilon_0 A} \qquad V = \left(\frac{mg \, 2 d^2}{\epsilon_r \epsilon_0 A} \right)^{1/2}$$

$$V = \left(\frac{10^{-2}(9.8)(2)(6 \times 10^{-3})^2}{(5)(8.85 \times 10^{-12})(5 \times 10^{-2})^2} \right)^{1/2}$$

$$= \left(\frac{7.06 \times 10^{-6}}{1.1 \times 10^{-13}} \right)^{1/2}$$

$$= (6.42 \times 10^{7})^{1/2} = 8.01 \ \text{kV}$$

$$\boxed{8 \ \text{kV}}$$

4) (25pts) These are short answer questions. Be specific, but **brief**.

If $\nabla \cdot \underline{J} \neq 0$ at a point, what must be happening there?

The amount of charge there must be changing in time

continuity eqn

Is the relationship $\underline{J} = \sigma\, \underline{E}$ always true?

No, not if we have a convection current

What does $\nabla \cdot \underline{B} = 0$ tell us about magnetic field lines?

no magnetic charges (sources or sinks of flux)

Field lines close on themselves

Write the expression for the point form of Ampere's Law

$\nabla \times \underline{H} = \underline{J}$ or $\nabla \times \underline{B} = \mu_0 \underline{J}$

How do we find the current passing through a contour "C"

$I = \iint_S \underline{J} \cdot d\underline{S}$ where S is a surface bounded by C

Give three expressions for the energy density in the electric field

$w_E = \frac{1}{2}\underline{E} \cdot \underline{D} = \frac{\epsilon \underline{E}^2}{2} = \frac{D^2}{2\epsilon}$

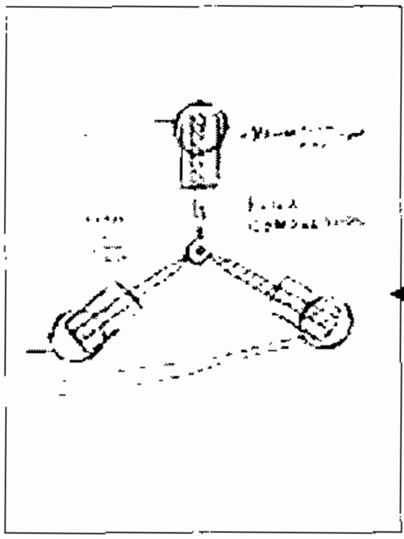
Give three expressions for the dissipated power density with a conduction current

$P_d = \underline{J} \cdot \underline{E} = \sigma E^2 = \frac{J^2}{\sigma}$

Write down the "similarity relation" or "duality of J and D : Resistance-Capacitance Analogy"

$RC = \epsilon / \sigma$

5) (10 points) Doc Brown finally realized his dream of the flux capacitor for powering his DeLorean time machine. He needed to figure out the capacitance between this and the rectangular box that was to enclose it. Having taken ECE 220, he filled the box containing the capacitor with salt water with a conductivity of 4 S/m and permittivity of ε_0. He then hooked up a power supply, voltmeter and ammeter and measured a current of 1kA when the applied voltage was 1 mV. What did he determine was the capacitance between his 'flux-capacitor' and its rectangular enclosure?



use similarity $RC = \epsilon / \sigma$ $\quad R = \frac{V}{I} = \frac{10^{-3}\text{ Volts}}{10^{3}\text{ Amps}} = 10^{-6}\,\Omega$

$$C = \frac{8.85 \times 10^{-12}}{4(10^{-6})} = \frac{8.85}{4} \times 10^{-6} = \underline{2.21\,\mu F}$$

Bonus Question (5 points): If we have a magnetic field given by $\underline{B} = 5\hat{x} + 3\hat{y}$ and an electric field given by $\underline{E} = 2\hat{z}$, in which direction does a charged particle drift? DRIFT in $\underline{E} \times \underline{B}$ direction

$$2\hat{z} \times (5\hat{x} + 3\hat{y}) = 10(\hat{z} \times \hat{x}) + 6(\hat{z} \times \hat{y})$$

$$= 10\hat{y} - 6\hat{x}$$

direction (unit vector) $= \frac{10\hat{y} - 6\hat{x}}{\sqrt{136}}$