ECE 220

CHANGE OF COORDINATE SYSTEMS

Cartesian to Cylindrical

Change of component

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Change of variable

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}(y / x)$$
$$z = z$$

Cylindrical to Cartesian

Change of component

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Change of variable

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

where: $\sin\phi = \dfrac{y}{\sqrt{x^2 + y^2}}$

$\cos\phi = \dfrac{x}{\sqrt{x^2 + y^2}}$

Cartesian to Spherical

Change of component

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Change of variable

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$
$$\phi = \tan^{-1}(y / x)$$

Spherical to Cartesian

Change of component

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

where: $\sin\theta = \dfrac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$ $\sin\phi = \dfrac{y}{\sqrt{x^2+y^2}}$

$\cos\theta = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$ $\cos\phi = \dfrac{x}{\sqrt{x^2+y^2}}$

Change of variable

$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

Cylindrical to Spherical

Change of component

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Change of variable

$r = \sqrt{\rho^2+z^2}$

$\theta = \cos^{-1}\left(\dfrac{z}{\sqrt{\rho^2+z^2}}\right)$

$\phi = \phi$

Spherical to Cylindrical

Change of component

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

where: $\sin\theta = \dfrac{\rho}{\sqrt{\rho^2+z^2}}, \cos\theta = \dfrac{z}{\sqrt{\rho^2+z^2}}$

Change of variable

$\rho = r\sin\theta$
$\phi = \phi$
$z = r\cos\theta$

BRIEF MATHEMATICAL TABLES FOR ECE 220

A. INTEGRALS

1. $$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = 1n(x + \sqrt{x^2 \pm a^2})$$

2. $$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

3. $$\int \frac{x^2\,dx}{\sqrt{x^2 \pm a^2}} = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2}\ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

4. $$\int \frac{dx}{(\sqrt{x^2 \pm a^2})^3} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

5. $$\int \frac{x\,dx}{(\sqrt{x^2 \pm a^2})^3} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

6. $$\int \frac{x^2\,dx}{(\sqrt{x^2 \pm a^2})^3} = \frac{-x}{\sqrt{x^2 \pm a^2}} + \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

7. $$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

8. $$\int \frac{x\,dx}{x^2 + a^2} = \frac{1}{2}\ln\left(x^2 + a^2\right)$$

9. $$\int \sin^n x\,dx = -\left[\left(\sin^{n-1} x \cos x\right)/\,n\right] + \frac{n-1}{n}\int \sin^{n-2} x\,dx, \qquad n > l$$

10. $$\int \cos^n x\,dx = \left[\left(\cos^{n-1} x \sin x\right)/\,n\right] + \frac{n-1}{n}\int \cos^{n-2} x\,dx, \qquad n > l$$

B. BINOMIAL SERIES:

$$(1 \pm x)^n = 1 \pm nx + \frac{1}{2!}n(n-1)x^2 \pm \frac{1}{3!}n(n-1)(n-2)x^3 + \cdots \qquad \text{for } x^2 < 1$$

$$(1 \pm x)^{-n} = 1 \mp nx + \frac{1}{2!}n(n+1)x^2 \mp \frac{1}{3!}n(n+1)(n+2)x^3 + \cdots \qquad \text{for } x^2 < 1$$

C. HYPERBOLIC IDENTITIES:

$$1n\left[\sqrt{x^2 + 1} + x\right] = \sinh^{-1}(x), \qquad 1n\left[\sqrt{x^2 - 1} + x\right] = \cosh^{-1}(x)$$