

## Homework 8

DUE                                      Friday, 14 March, 8:50 am  
Read                                      Ch. 8 (intro: p. 231-232), Ch. 9 (sections 1, 2, 7, 8; skim 3-5), *BSL*

1. The mechanical energy balance contains the following kinetic energy term

$$\frac{1}{2} \frac{\langle \bar{v}^3 \rangle}{\langle \bar{v} \rangle}$$

In turbulent flow, we argued that because the velocity profile is relatively flat ('plug flow') one may make the approximation  $\langle \bar{v}^3 \rangle \approx \langle \bar{v} \rangle^3$ , which then allows the kinetic energy term to be simplified. More generally, the kinetic energy term is often rewritten in the form

$$\frac{1}{2} \frac{\langle \bar{v}^3 \rangle}{\langle \bar{v} \rangle} = \frac{1}{2\alpha} \langle \bar{v} \rangle^2$$

where  $\alpha$  is a function of the Reynolds number (Re). Using the velocity profiles given below, evaluate the averages to solve for  $\alpha$ .

(a) laminar flow in a tube where the velocity profile is given as

$$v_z(r) = v_{z,\max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

(b) turbulent flow in a tube with  $10^4 < \text{Re} < 10^5$ , where the time-averaged velocity profile is approximately

$$\bar{v}_z(r) = \bar{v}_{z,\max} \left[ 1 - \frac{r}{R} \right]^{1/7}$$

(Thanks to Prof. Klingenberg for this problem.)

2. Solve Problem 7A.2 Pumping a hydrochloric acid solution.
3. Solve Problem 7A.3 Compressible gas flow in a cylindrical pipe .
4. Solve Problem 7B.3 Flow through a sudden enlargement.
5. Solve Problem 7.B.4 Flow between two tanks