

Homework 15 (last!)

DUE Friday, 9 May, 8:50 am
Read Ch 18 (sec 7), Ch. 19 (1-3, 5), Ch. 20 (sec 1)
 Ch 22 (sec 1-3)

Note: The Final Exam will be Monday, May 12 at 12:25pm in 3024 (last name A-K) and 3032 (last name L-Z) Engineering Hall. The exam will be closed-book and closed-note.

1. Review of Exam 3. This exercise aims to help you learn from errors or problems that may have arisen during exam 3. It applies to every problem for which you did not receive full credit.
 - (a) If you lost points due to a mathematical error, explain what the error was and suggest a strategy to help you avoid similar errors in the future.
 - (b) If you lost full credit for other reasons, then solve the full problem here. If you are unable to get a start on the solution, study the solution key. Then, without referring to the solution key, re-read the problem and solve it on your own. DO NOT simply copy the solution from the key or you defeat the purpose of this exercise.

All reworked exam solutions (parts a and b) should accompany your Homework 15. After completing this exercise you should be able to re-take Exam 3 and achieve a perfect score.

2. Solve Problem 18B.2 Error in neglecting the convection term in evaporation. (Note that part c refers to Example 18.2-2, not 18.2-1.)
3. Assuming we can define a characteristic length scale (L) and time scale (τ) for a problem, rewrite Fick's 2nd law of diffusion (Eq. 19.1-18) in dimensionless form and collect the single dimensionless group of characteristic parameters to the left-hand side of the equation.
 - (a) What is the dimensionless group?
 - (b) When the dimensionless group is of magnitude unity, one may describe the system as "significantly advanced," enabling us to very roughly estimate a length scale that a diffusion front has penetrated, given an elapsed time and diffusivity. Notice that in solving the analogous unsteady momentum and energy transfer problems (Examples 4.1-1 and 12.1-1, respectively) a specific combination of variables was suggested as a means to convert the PDE into an ODE. By analogy with these solved cases, how might you propose to define a more precise penetration depth for diffusion?
4. In class we used dimensional analysis to explore the behavior of an artificial kidney. We sought to understand how the mass transfer coefficient for a toxin-permeable membrane might depend on fluid, toxin, flow and geometry, specifically:

$$k = C v^\alpha \rho^\beta \mu^\gamma D^\delta d^\epsilon$$

where k is the mass transfer coefficient of the toxin in blood, and system variables are velocity of blood flow, density and viscosity of blood, diffusivity of toxin in blood, and diameter of tubular membrane, respectively. By expressing each variable in terms of dimensions (mass, length, time) we obtained three constraints on the five unknowns (α , β , γ , δ , ϵ). We chose in class to group variables to



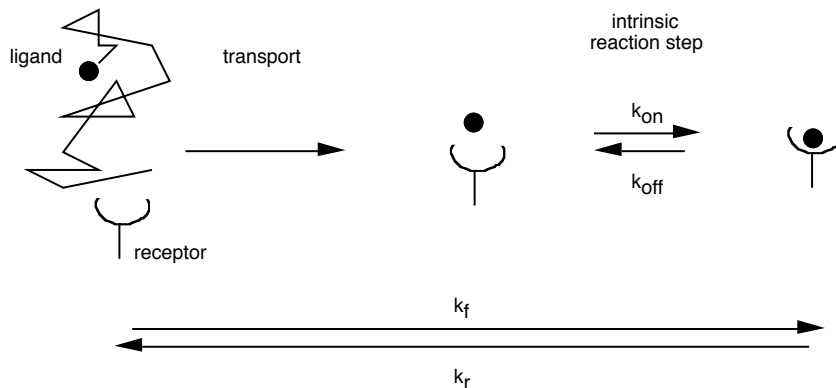
eliminate all but two unknowns (α and δ), allowing us to describe the dimensionless mass transfer coefficient (Sh number) in terms of Re and Sc numbers. However, any two unknowns would provide an equivalent expression. Show that this is true by grouping variables to express all unknowns in terms of (α and β) rather than (α and δ); do any dimensionless groups (such as Re or Sc) appear from this analysis? Using your solution (in terms of α and β) and show how the dependence of Sh on Re and Sc can be recovered through all little algebra.

5. The following problem was adapted from Lauffenburger and Linderman, *Receptors: Models for Binding, Trafficking, and Signaling*, Oxford University Press, 1993. **Diffusion coupled with reaction.** To coordinate functions such as tissue growth, metabolism, or defense against parasites, the cells of an organism communicate with each other by sending and receiving signals. One way cells send signals to neighboring cells is to secrete molecules or ‘ligands,’ that diffuse to and then bind to ‘receptor’ molecules present on the surface of nearby cells. Clinicians or tissue engineers may seek to modulate cell-cell communications by perturbing the normal ligand-receptor interactions. For example, by providing freely soluble receptor one can competitively bind free ligands, making them unavailable for binding to cell-associated receptors.

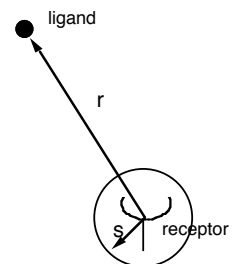
Consider the rate of reversible binding of ligand (L) to receptor (R) when both are free in solution:



With association rate constant k_f (forward) and dissociation rate constant k_r (reverse). These rate constants combine both effects of transport and intrinsic binding, as shown in the schematic below:



(a) Derive an expression for $L(r)$, the ligand concentration as a function of r , where r is the distance from a single receptor molecule located at the origin of a spherical coordinate system. Assume transport occurs by diffusion with diffusion constant $D (= D_L + D_R)$. A schematic of the geometry is shown to the right. Here s is the ‘encounter radius’ or the separation distance between the ligand and receptor where they are close enough to bind. Assume that the concentration of ligand is L_o at $r = \infty$ and the diffusive flux multiplied by the area at $r = s$ is equivalent to $k_{on} L(s)$.



(b) If the overall flux of ligand molecular to the receptor is given by $k_f L_o$, and this is equal to the diffusive flux at $r = s$, then derive an expression for k_f in terms of k_{on} and k_{trans} , where $k_{trans} = 4\pi Ds$.