8

Adaptive Filters

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This chapter discusses how to build adaptive digital filters to perform noise cancellation and signal extraction. Adaptive techniques are advantageous because they do not require a priori knowledge of the signal or noise characteristics as do fixed filters. Adaptive filters employ a method of learning through an estimated synthesis of a desired signal and error feedback to modify the filter parameters. Adaptive techniques have been used in filtering of 60-Hz line frequency noise from ECG signals, extracting fetal ECG signals, and enhancing P waves, as well as for removing other artifacts from the ECG signal. This chapter provides the basic principles of adaptive digital filtering and demonstrates some direct applications.

In digital signal processing applications, frequently a desired signal is corrupted by interfering noise. In fixed filter methods, the basic premise behind optimal filtering is that we must have knowledge of both the signal and noise characteristics. It is also generally assumed that the statistics of both sources are well behaved or wide-sense stationary. An adaptive filter learns the statistics of the input sources and tracks them if they vary slowly.

8.1 PRINCIPAL NOISE CANCELER MODEL

In biomedical signal processing, adaptive techniques are valuable for eliminating noise interference. Figure 8.1 shows a general model of an adaptive filter noise canceler. In the discrete time case, we can model the primary input as $s(nT) + n_0(nT)$. The noise is additive and considered uncorrelated with the signal source. A secondary reference input to the filter feeds a noise $n_1(nT)$ into the filter to produce output $\zeta(nT)$ that is a close estimate of $n_0(nT)$. The noise $n_1(nT)$ is correlated in an unknown way to $n_0(nT)$.

The output $\zeta(nT)$ is subtracted from the primary input to produce the system output $y(nT)$. This output is also the error $\varepsilon(nT)$ that is used to adjust the taps of the adaptive filter coefficients $\{w(1,\ldots, p)\}$.

$$y(nT) = s(nT) + n_0(nT) - \zeta(nT) \tag{8.1}$$

Squaring the output and making the (nT) implicit to simplify each term

$$y^2 = s^2 + (n_0 - \zeta)^2 + 2s(n_0 - \zeta) \tag{8.2}$$

Taking the expectation of both sides,

$$\begin{aligned} E[y^2] &= E[s^2] + E[(n_0 - \zeta)^2] + 2E[s(n_0 - \zeta)] \\ &= E[s^2] + E[(n_0 - \zeta)^2] \end{aligned} \tag{8.3}$$

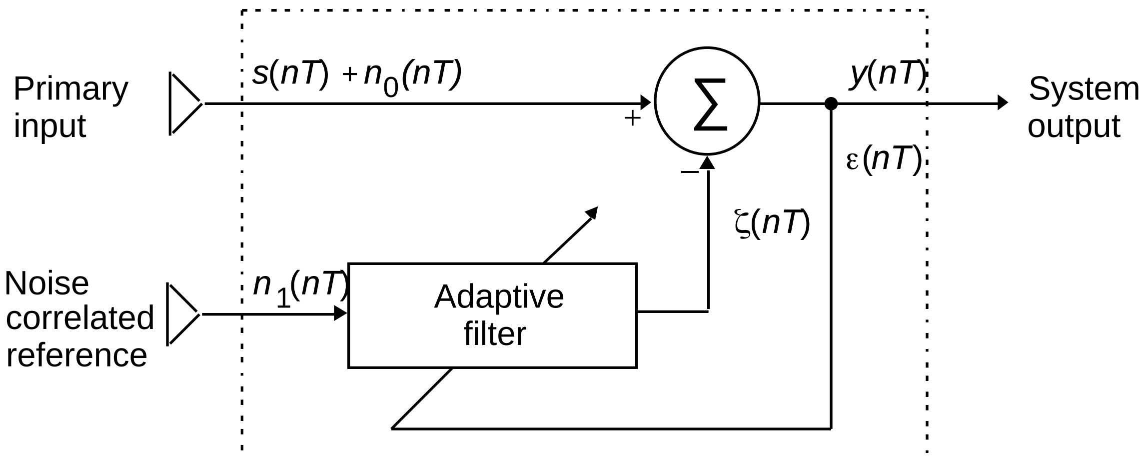

Figure 8.1 The structure of an adaptive filter noise canceler.

Since the signal power $E[s^2]$ is unaffected by adjustments to the filter

$$\min E[y^2] = E[s^2] + \min E[(n_0 - \zeta)^2] \tag{8.4}$$

When the system output power is minimized according to Eq. (8.4), the mean-squared error (MSE) of $(n_0 - \zeta)$ is minimum, and the filter has adaptively learned to synthesize the noise ($\zeta \approx n_0$). This approach of iteratively modifying the filter coefficients using the MSE is called the Least Mean Squared (LMS) algorithm.

8.2 60-HZ ADAPTIVE CANCELING USING A SINE WAVE MODEL

It is well documented that ECG amplifiers are corrupted by a sinusoidal 60-Hz line frequency noise (Huhta and Webster, 1973). As discussed in Chapter 5, a non-recursive band-reject notch filter can be implemented to reduce the power of noise at 60 Hz. The drawbacks to this design are that, while output noise power is reduced, such a filter (1) also removes the 60-Hz component of the signal, (2) has a very slow rolloff that unnecessarily attenuates other frequency bands, and (3) becomes nonoptimal if either the amplitude or the frequency characteristics of the noise change. Adaptive transversal (tapped delay line) filters allow for elimination of noise while maintaining an optimal signal-to-noise ratio for nonstationary processes.

One simplified method for removal of 60-Hz noise is to model the reference source as a 60-Hz sine wave (Ahlstrom and Tompkins, 1985). The only adaptive parameter is the amplitude of the sine wave. Figure 8.2 shows three signals: $x(nT)$ is the input ECG signal corrupted with 60-Hz noise, $e(nT)$ is the estimation of the noise using a 60-Hz sine wave, and $y(nT)$ is the output of the filter.

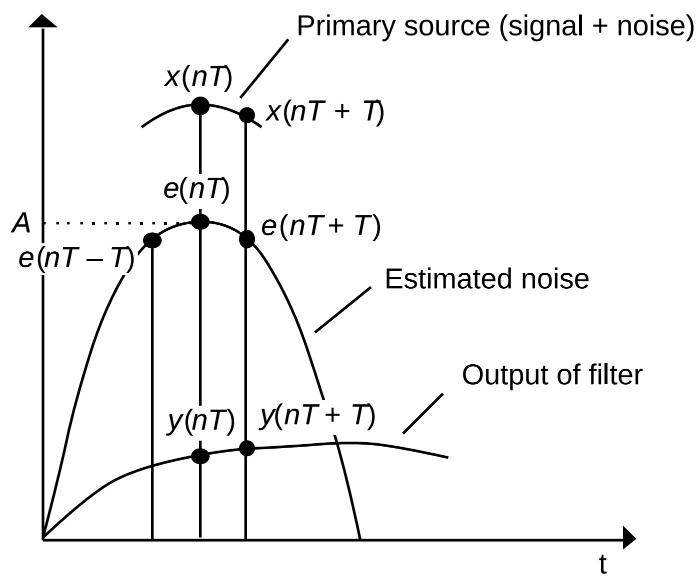

Figure 8.2 Sine wave model for 60-Hz adaptive cancellation.

The algorithm begins by estimating the noise as an assumed sinusoid with amplitude A and frequency ω

$$e(nT) = A\sin(\omega nT) \tag{8.5}$$

In this equation, we replace term (nT) by $(nT - T)$ to find an expression for the estimated signal one period in the past. This substitution gives

$$e(nT - T) = A\sin(\omega nT - \omega T) \tag{8.6}$$

Similarly, an expression that estimates the next point in the future is obtained by replacing (nT) by $(nT + T)$ in Eq. (8.5), giving

$$e(nT + T) = A\sin(\omega nT + \omega T) \tag{8.7}$$

We now recall a trigonometric identity

$$\sin(\alpha + \beta) = 2\sin(\alpha)\cos(\beta) - \sin(\alpha - \beta) \tag{8.8}$$

Now let

$$\alpha = \omega nT \quad \text{and} \quad \beta = \omega T \tag{8.9}$$

Expanding the estimate for the future estimate of Eq. (8.7) using Eqs. (8.8) and (8.9) gives

$$e(nT + T) = 2\,\underline{A\sin(\omega nT)}\;\cos(\omega T) - \underline{A\sin(\omega nT - \omega T)} \tag{8.10}$$

Note that the first underlined term is the same as the expression for $e(nT)$ in Eq. (8.5), and the second underlined term is the same as the expression for $e(nT - T)$ in Eq. (8.6). The term, $\cos(\omega T)$, is a constant determined by the frequency of the noise ω to be eliminated and by the sampling frequency, $f_s = 1/T$:

$$N = \cos(\omega T) = \cos\left(\frac{2\pi f}{f_s}\right) \tag{8.11}$$

Thus, Eq. (8.10) is rewritten, giving a relation for the future estimated point on a sampled sinusoidal noise waveform based on the values at the current and past sample times.

$$e(nT + T) = 2Ne(nT) - e(nT - T) \tag{8.12}$$

The output of the filter is the difference between the input and the estimated signals

$$y(nT + T) = x(nT + T) - e(nT + T) \tag{8.13}$$

Thus, if the input were only noise and the estimate were exactly tracking (i.e., modeling) it, the output would be zero. If an ECG were superimposed on the input noise, it would appear noise-free at the output.

The ECG signal is actually treated as a transient, while the filter iteratively attempts to change the "weight" or amplitude of the reference input to match the desired signal, the 60-Hz noise. The filter essentially learns the amount of noise that is present in the primary input and subtracts it out. In order to iteratively adjust the filter to adapt to changes in the noise signal, we need feedback to adjust the sinusoidal amplitude of the estimate signal for each sample period.

We define the difference function

$$f(nT + T) = [x(nT + T) - e(nT + T)] - [x(nT) - e(nT)] \tag{8.14}$$

In order to understand this function, consider Figure 8.3. Our original model of the noise $e(nT)$ in Eq. (8.5) assumed a simple sine wave with no dc component as shown. Typically, however, there is a dc offset represented by V_{dc} in the input $x(nT)$ signal. From the figure

$$V_{dc}(nT+T) = x(nT+T) - e(nT+T) \tag{8.15}$$

and also

$$V_{dc}(nT) = x(nT) - e(nT) \tag{8.16}$$

Assuming that the dc level does not change significantly between samples, then

$$V_{dc}(nT+T) - V_{dc}(nT) = 0 \tag{8.17}$$

This subtraction of the terms representing the dc level in Eqs. (8.15) and (8.16) is the basis for the function in Eq. (8.14). It subtracts the dc while simultaneously comparing the input and estimated waveforms.

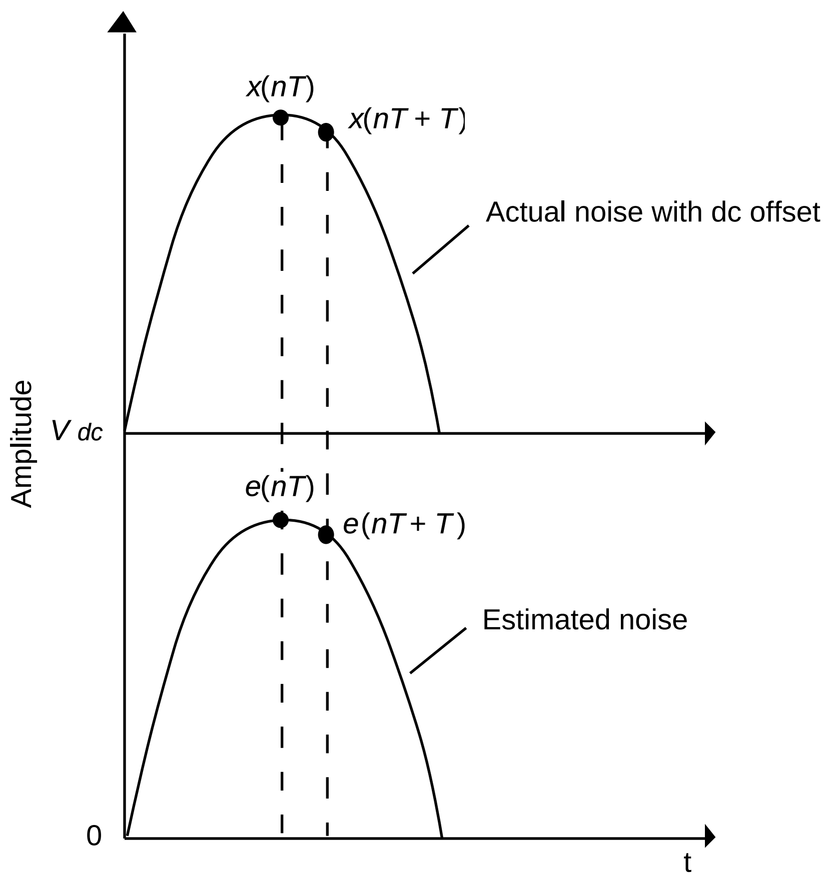

Figure 8.3 The actual noise waveform may include a dc offset that was included in the original model of the estimated signal.

We use $f(nT + T)$ to determine if the estimate $e(nT)$ was too large or too small. If $f(nT + T) = 0$, the estimate is correct and there is no need to adjust the future estimate, or

$$e(nT+T) = e(nT+T) \tag{8.18}$$

If $f(nT + T) > 0$, the estimate is low, and the estimate is adjusted upward by a small step size d

$$e(nT + T) = e(nT + T) + d \tag{8.19}$$

If $f(nT + T) < 0$, the estimate is high and the estimate is adjusted downward by a small step size d

$$e(nT + T) = e(nT + T) - d \tag{8.20}$$

The choice of d is empirically determined and depends on how quickly the filter needs to adapt to changes in the interfering noise. If d is large, then the filter quickly adjusts its coefficients after the onset of 60-Hz noise. However, if d is too large, the filter will not be able to converge exactly to the noise. This results in small oscillations in the estimated signal once the correct amplitude has been found. With a smaller d, the filter requires a longer learning period but provides more exact tracking of the noise for a smoother output. If the value of d is too large or too small, the filter will never converge to a proper noise estimate.

A typical value of d is less than the least significant bit value of the integers used to represent a signal. For example, if the full range of numbers from an 8-bit A/D converter is 0–255, then an optimal value for d might be 1/4.

Producing the estimated signal of Eq. (8.12) requires multiplication by a fraction N given in Eq. (8.11). For a sampling rate of 500 sps and 60-Hz power line noise

$$N = \cos\left(\frac{2\pi \times 60}{500}\right) = 0.7289686 \tag{8.21}$$

Such a multiplier requires floating-point arithmetic, which could considerably slow down the algorithm. In order to approximate such a multiplier, we might choose to use a summation of power-of-two fractions, which could be implemented with bit-shift operations and may be faster than floating-point multiplication in some hardware environments. In this case

$$N = \frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{128} + \frac{1}{512} + \frac{1}{2048} = 0.72900 \tag{8.22}$$

8.3 OTHER APPLICATIONS OF ADAPTIVE FILTERING

Adaptive filtering is not only used to suppress 60-Hz interference but also for signal extraction and artifact cancellation. The adaptive technique is advantageous for generating a desired signal from one that is uncorrelated with it.

8.3.1 Maternal ECG in fetal ECG

Prenatal monitoring has made it possible to detect the heartbeat of the unborn child noninvasively. However, motion artifact and the maternal ECG make it very difficult to perceive the fetal ECG since it is a low-amplitude signal. Adaptive filtering has been used to eliminate the maternal ECG. Zhou et al. (1985) describe an algorithm that uses a windowed LMS routine to adapt the tap weights. The abdominal lead serves as the primary input and the chest lead from the mother is used as the reference noise input. Subtracting the best matched maternal ECG from the abdominal ECG which contains both the fetal and maternal ECGs produces a residual signal that is the fetal ECG.

8.3.2 Cardiogenic artifact

The area of electrical impedance pneumography has used adaptive filtering to solve the problem of cardiogenic artifact (ZCG). Such artifact can arise from electrical impedance changes due to blood flow and heart-volume changes. This can lead to a false interpretation of breathing. When monitoring for infant apnea, this might result in a failure to alarm. Sahakian and Kuo (1985) proposed using an adaptive LMS algorithm to extract the cardiogenic impedance component so as to achieve the best estimate of the respiratory impedance component. To model the cardiogenic artifact, they created a template synchronized to the QRS complex in an ECG that included sinus arrhythmia. Cardiogenic artifact is synchronous with but delayed from ventricular systole, so the ECG template can be used to derive and eliminate the ZCG.

8.3.3 Detection of ventricular fibrillation and tachycardia

Ventricular fibrillation detection has generally used frequency-domain techniques. This is computationally expensive and cannot always be implemented in real time. Hamilton and Tompkins (1987) describe a unique method of adaptive filtering to locate the poles corresponding to the frequency spectrum formants. By running a second-order IIR filter, the poles derived from the coefficients give a fairly good estimate of the first frequency peak.

The corresponding z transform of such a filter is

$$H(z) = \frac{1}{1 - b_1 z^{-1} - b_2 z^{-2}}$$

We can solve for the pole radius and angle by noting that, for all poles not on the real axis

$$b_1 = 2r\cos\theta \quad \text{and} \quad b_2 = -r^2$$

Using the fact that fibrillation produces a prominent peak in the 3–7 Hz frequency band, we can determine whether the poles fall in the “detection region” of the z plane. An LMS algorithm updates the coefficients of the filter. Figure 8.4 shows the z-plane pole-zero diagram of the adaptive filter. The shaded region indicates that the primary peak in the frequency spectrum of the ECG is in a “dangerous” area. The only weakness of the algorithm is that it creates false detections for rhythm rates greater than 100 bpm with frequent PVCs, atrial fibrillation, and severe motion artifact.

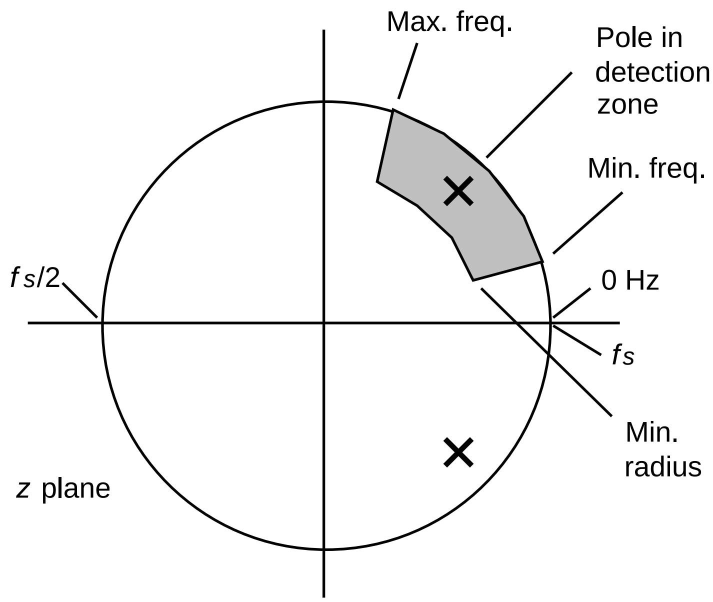

Figure 8.4 The *z* plane showing the complex-conjugate poles of the second-order adaptive filter.

8.4 LAB: 60-HZ ADAPTIVE FILTER

Load UW DigiScope, select **`ad(V) Ops`**, then **`(A)daptive`**. This module is a demonstration of a 60-Hz canceling adaptive filter as described in the text. You have control over the filter's step size *d*. This controls how quickly the filter *learns* the amount of 60 Hz in the signal. By turning the 60-Hz noise off after the filter has adapted out the noise, you can observe that the filter must now unlearn the 60-Hz component. This routine always uses the same data file **`adapting.dat`** to which the 60-Hz noise is added.

8.5 REFERENCES

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8.6 STUDY QUESTIONS

8.1 What are the main advantages of adaptive filters over fixed filters?

8.2 Explain the criterion that is used to construct a Wiener filter.

8.3 Why is the error residual of a Wiener filter normal to the output?

8.4 Design an adaptive filter using the method of steepest-descent.

8.5 Design an adaptive filter using the LMS algorithm.

8.6 Why are bounds necessary on the step size of the steepest-descent and LMS algorithms?

8.7 What are the costs and benefits of using different step sizes in the 60-Hz sine wave algorithm?

8.8 Explain how the 60-Hz sine wave algorithm adapts to the phase of the noise.

8.9 The adaptive 60-Hz filter calculates a function

$$f(nT + T) = [x(nT + T) - e(nT + T)] - [x(nT) - e(nT)]$$

If this function is less than zero, how does the algorithm adjust the future estimate, $e(nT + T)$?

8.10 The adaptive 60-Hz filter uses the following equation to estimate the noise:

$$e(nT + T) = 2Ne(nT) - e(nT - T)$$

If the future estimate is found to be too high, what adjustment is made to (a) $e(nT - T)$, (b) $e(nT + T)$. (c) Write the equation for N and explain the terms of the equation.

8.11 The adaptive 60-Hz filter calculates the function

$$f(nT + T) = [x(nT + T) - e(nT + T)] - [x(nT) - e(nT)]$$

It adjusts the future estimate $e(nT + T)$ based on whether this function is greater than, less than, or equal to zero. Use a drawing and explain why the function could not be simplified to

$$f(nT + T) = x(nT + T) - e(nT + T)$$